Distributed Control of a Network with Multiple Electricity Producers and Consumers

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Gunn Kristine Holst Larsen

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Foar Auke
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Contents

Acknowledgements **ix**

Notations **xiii**

Acronyms **xv**

1 Introduction **1**
1.1 Matching Power Supply and Demand 1
1.2 Contributions and Outline of the Thesis 7
1.3 The Flexines Project 9
1.4 Ongoing Work 11

2 Optimal Control Preliminaries **13**
2.1 Optimization and Duality 13
2.2 Optimal Control in a Network of Decision Makers 18
2.3 Model Predictive Control (MPC) 24
2.4 Discussion 31

3 A New Information Sharing Model for the Smart Grid **33**
3.1 Problem Setting 33
3.2 Information Network Model 35
3.3 Constructing the Information Matrix 41
3.4 Discussion 45

4 Control of a Network with Multiple Electricity Producers **47**
4.1 Problem Setting 47
4.2 Prosumers with Unconstrained Electricity Production 49

4.3 Prosumers with μ-CHPs Including on-off Constraints and Power Modulation 55

5 Prosumers with μ-CHPs Including Heat Buffers 69
5.1 Problem Setting 69
5.2 System Description and Problem Formulation 71
5.3 Results 78
5.4 Discussion 87

6 Control of a Network with Multiple Electricity Consumers 89
6.1 Heavy Demand Consumers in a Network 90
6.2 Results 93
6.3 Discussion 99

7 Conclusion and Outlook 101
7.1 Conclusions 101
7.2 Future Work 105

A Complementary Data 109

Bibliography 117

Summary 123

Samenvatting 127

About the Author 131

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Gunn Kristine Holst Larsen,
Groningen, January 2014.

Notations

The following notations are used throughout the thesis.

Notation	Meaning
$\mathbb{R}$	The real numbers
$\mathbb{R}^n$	The vector space of n-component real vectors
$\mathbb{R}^{n\times m}$	The vector space of $n\times m$ real matrices
$x \geq y$	For $x,y\in\mathbb{R}^n$ we have $x_i \geq y_i$ for all $i=1,\dots,n$
$x > y$	For $x,y\in\mathbb{R}^n$ we have $x_i > y_i$ for all $i=1,\dots,n$
$\mathbb{R}_+$	The positive real numbers including zero $\{x\in\mathbb{R}^n : x\geq 0\}$
$\mathbb{R}_-$	The negative real numbers including zero $\{x\in\mathbb{R}^n : x\leq 0\}$
x^T	The transpose of a vector x
$\lvert x(k)\rvert_Q^2$	The product $x^\mathsf{T}(k)Qx(k)$
$\limsup$	The asymptotic supremum of a function
$\{x(k)\}_{k=0}^{k=\infty}$	The sequence $x(0), x(1), x(2), \dots$
$R>0$	The matrix R is positive-definite
$R\geq 0$	The matrix R is positive-semidefinite
$diag(L)$	The matrix $\bar{L}=diag(L)\in\mathbb{R}^{n\times n}$ has the same diagonal as the matrix $L\in\mathbb{R}^{n\times n}$, $\bar{L}_{ii}=L_{ii}$, and it is zero elsewhere, $\bar{L}_{ij}=0$ if $j\neq i$.
$\mathbf{E}X$	The expected value of the random variable X

Acronyms

The following acronyms are used throughout the thesis.

Acronym	Meaning
μ-CHP	Micro combined heat and power
MPC	Model predictive control
d-MPC	Distributed model predictive control
LQR	Linear quadratic regulator
QP	Quadratic program
MIQP	Mixed integer quadratic program
OPF	Optimal power flow
TSO	Transmission system operator
DSO	Distribution system operator

CHAPTER 1

Introduction

This thesis is concerned with distributed control of multiple electricity producers and consumers connected in a network. We develop an information sharing model, and combine it with a distributed Model Predictive Control algorithm in case studies with realistic data. This chapter introduces the problem setting, highlights some related smart grid research, provides the list of main contributions, and finishes with the outline of the thesis.

1.1 Matching Power Supply and Demand

A well functioning electrical power grid is a key ingredient for supporting our modern society. Factories need it for producing goods; hospitals depend on electrical equipment to save lives; and domestic appliances, such as dish washers and TV's that improve the comfort-level of our lives, run on electricity. We take it for granted that the power grid is available and reliable, but behind the scene there is a highly complex system that integrates technology, economics and politics.

Electric power itself is nothing but electric charges that flow in a conducting material due to an electrical potential. This electric current can be used to energize equipment. Typically, the distance from the power plants, where the power is generated, to the consumers is large. For the power grid, which connects the power suppliers and the power consumers, these large distances introduce problems. One problem is that energy is lost in the transmission lines, and a longer distance between the supplier and consumer implies a larger energy loss. However, at higher voltages the same amount of energy can be transferred at a lower current. In this case, less energy is lost. As a result, today's power grid has a layered structure with respect to voltages in the power lines. After the power is generated, the power is transformed up to high voltages to be transmitted in the transmission lines over long distances. At demand stations, the power is transformed down again to suitable voltages for the distribution network where the customers are connected.

The voltage and frequency in the power grid will rise if the power production exceeds the power consumption and vice versa. However, domestic equipment is designed to work on a nominal voltage and frequency, which in the case of The Netherlands are 220V and 50Hz. Therefore, it is important to balance the supply and demand at any time, such that the voltage and frequency stay at their target values.

The end-users of electricity are currently passive, i.e. i their electricity production and consumption are not influenced by for example real-time prices. Thus, to achieve a power balance a few central power generators are controlled to meet the demand from a large number of end-users. In fact, the control loops to ensure the balance are implemented in a hierarchical fashion. The control layers include day ahead planning as well as real-time balancing, because electricity cannot be stored efficiently in large quantities. At the same time, all business components in the power system should be allowed to compete fairly and freely, see e.g. Amin [1]. It is clear that the power system is highly complex and difficult to control.

Today's power grid systems have functioned well for decades, but we have reached a point in time where we expect that changes cannot be avoided. One trend that requires change is the growing environmental awareness. Roughly speaking 10% of the energy is lost in the power lines in the current power grid, see e.g. Baum [2]. As electric power is mainly generated by fossil fuel, the following questions arise; 1) Can we use the resources more efficiently? 2) Can we replace the fossil resources by more environmentally friendly resources?

Over the last decade, climate changes have forced governments all over the world to put environmental questions on the agenda. In the Netherlands there are three major climate targets for 2020 with respect to the 1990 levels; reduce the greenhouse gas emissions with 20%, increase the share of renewable sources in final energy consumption to 20%, and obtain a 20% increase in energy efficiency, c.f. Cramer [12]. A consequence of an increasing share of renewable sources like wind and solar in the electricity mixture, is that decentralized generation becomes a more important part of the power system due to the nature of these sources. The locations of wind and solar farms are often geographically distributed. Another aspect is that the generation from these sources is heavily depended on uncontrollable weather conditions. Taking the weather conditions into account, the surplus or shortage of electricity production needs to be balanced with alternative generators. This may make it even more difficult for the conventional power plants to balance the network.

Another trend is the never ending increased electrification of the society. For example the number of electric cars on the road is increasing. Suppose that all the electric cars charge their batteries at the same time. Then this would result in a peak in the power load. This challenges the capacity in the network, as each power line has a maximum capacity. The power network is designed so as to handle the largest peak load during a day, and updating the grid so that it can handle a higher load is costly. It would be better for the power network if some of the load could be shifted in time, so that the load is flattened and the network could handle more connections on the current infrastructure.

Traditionally only power generation is seen as a flexible variable that can be subject to control. However, there is also flexibility in the demand if the end-users get control incentives. This is known as *demand response*, and the incentives can for example be given through price information. Demand response is treated in Chapter 6. Notice that in the management science literature, demand response is called demand management, see e.g. [59].

Controllable domestic generators offer a solution to the line losses problem, as well as it can help fill in the uncontrollable fluctuations of renewable energy sources. One example of a controllable domestic generator is the micro Combined Heat and Power (μ-CHP) system, which can produce heat and power in a household. By producing power locally, the losses in the transportation lines are avoided, and by using the heat output there is no waste heat. Typically, μ-CHP systems run on gas, which make them particularly interesting to install in households in countries like the Netherlands where the gas grid is dense, see Van der Veen [63]. A network of μ-CHPs is treated in Chapters 4 - 5.

Both demand response and distributed generation are parts of the *Smart Grid*, which is expected to be the future power network, see e.g. Khattak et al. [31]. Here the term "smart grid" reflects that there is local intelligence present throughout the network. Computers, communication, sensing and control technology operate in parallel with the electric power grid to achieve several goals. Examples of such goals are the reliability of the electric power delivery, the minimization of the cost of electric energy to consumers, and the facilitation of the interconnection of new generation sources, see e.g. Amin [1]. The Smart Grid offers a number of significant advantages. First, the Smart Grid allows for two-way communication, which enables demand response. Secondly, domestic power generation is a key component, which makes the end-user both a producer and a consumer, or a *prosumer*, of electric power. In a Smart Grid, prosumers are both incentivized and empowered to contribute to the balance of power supply and demand in the power system. Thirdly, by producing and consuming power locally, Smart Grids also minimize transportation losses; a feature of the Smart Grid which offers both economic and environmental gains.

Another important feature of the Smart Grid is found in the fact that local matching can lower the fluctuations in power flow over the transformer stations in the power system. Smart Grids can therefore ease the control efforts to achieve a power balance in the overall power system, see e.g. Tekier-Mogulkoc et al. [61]. However, because each end-user decides when to use his electric devices, a major question that arises here is: how do we coordinate the decisions of a large number of end-users to benefit the power system? In the power system, the end-users can have a large variety of electric power demand and production devices, such as washing machines, freezers and μ-CHP systems, which can be controlled even if they are subject to operational constraints. The rest of the power demand, that cannot be controlled, can, to some extent, be predicted. This means that the decisions on when to use the controllable devices have to be coordinated on two levels. Firstly, the end-user has to anticipate on the forecasted power demand-production profile. Secondly, since electric power is shared in the power network, the end-user must also anticipate on how neighbors decisions influence the power profile. Therefore, in order for the end-users to contribute to the system in an optimal way, an *optimal control problem* has to be solved to coordinate the decisions.

As an example, suppose one household turns on the washing machine, but it cannot compensate for the increase in demand itself. The household is not able to turn off another electrical device or ramp up his power production. Then, to keep the balance in the network, the household might buy power from a neighbor. The neighbor can for example turn off its freezer and postpone its dish washer. As the network grows in the number of end-users, a large number of decision variables have to be included in the optimal control problem. The question to be investigated in this thesis is, therefore, how do we achieve coordination in the network in a scalable and efficient way? Smart Grid research is increasingly popular in energy companies, government institutions and universities around the world. In this section, we can only mention a very small selection of smart grid related work present in the literature.

A large body of research on power systems is related to the Optimal Power Flow (OPF) problem. The OPF problem is solved to find the optimal power generation given the line power constraints, see e.g. Dommel et al. [13], Stott [58], and Huneault et al. [28]. The objective of the OPF is to minimize generation cost while the balance problem is included as a hard constraint. It is a steady state optimal control problem. In Jokic [29], a dynamic distributed feedback controller for an optimal real-time update was designed. This is a price based optimal control of electrical power systems, and the controller reacts on the network frequency deviation as a measure of power imbalance in the system. However, predictions and anticipation on the future situation in the network cannot easily be included in a distributed setting.

In contrast to the OPF problem, where the power generation from large power plants is considered, we will work on a smaller scale, i.e. we work at a household level to coordinate electrical devices. All households connected in the same low voltage network, will in principle measure the same frequency deviation. Therefore, we will here base ourselves on communication between neighbors rather than frequency measurement while coordinating decisions in the Smart Grid. It is widely agreed that a centralized solution scheme for the so-called *optimal control problem* is too time consuming, because of the computational complexity, as pointed out in Kezunovic et al. [30]. Therefore, a host of scalable control methods have been proposed in the Smart Grid setting. Current methods, proposed by the literature for device coordination, have a centralized nature in the fact that there is one decision-making agent, see e.g. Molderink et al. [45]. In [45] a methodology combining forecasts, planning and real-time control is described. However, the planning is done in a centralized way. Another example is the PowerMatcher game which is presented in Kok et al. [32]. Here an agent for each device broadcasts a bidding curve for his willingness to pay for electricity. One agent at the top of a hierarchical structure then determines the equilibrium price. The PowerMatching concept was implemented in Groningen, in The Netherlands, as a demonstration project of a future energy-infrastructure called PowerMatching city[1]. Twenty-five households with smart appliances, such as μ-CHP systems that match their energy use in

[1] http://www.powermatchingcity.nl

real-time based upon the available energy generation, are connected. The project is generally perceived as a success. However, a number of short-comings were observed. In particular, predictions are not yet taken into account, and since the prices are the same everywhere in the network, there is no preferred location for the production in the network.

This observation motivates us to consider a network model with a distributed information structure, which will allow for the implementation of a scalable optimization method, as well as to include predictions on the power demand and supply. In order to anticipate on the future situation in the power network, and to incorporate physical constraints from the electric devices, we work in a Model Predictive Control (MPC) framework. In the MPC framework we include predictions about the end-users' future power demand, and technical constraints from the devices that need to be controlled, see e.g. Morari et al. [46], Mayne et al. [44], and Camacho et al. [8].

Probably the most related work to the results presented in this thesis is the work presented in Negenborn [48], where a multi-agent MPC approach is presented. However, there the method is applied to load frequency control, which is a different type of problem than the power balance at a market level problem that we treat and, most importantly, an information sharing structure has not been considered. In particular, we address the challenge to match local supply and demand in real-time anticipating on the future behavior and only base decisions on local information. By local information we mean the forecast at the agent itself, and information from directly connected neighbors in an information network. In general, distributed grids are more robust to topological failures, see Rohden et al. [54]. Thus, we can expect a benefit with respect to a centralized network in terms of robustness, power delivery reliability, computational scalability, and ultimately in the cost for the end-user.

We propose an information sharing network where all agents only have local (power imbalance) information about the system when they make their decisions to turn on and off electrical devices. The agents in the information network are an isolated subset of the agents in the power network. In a large network, the distance between suppliers and consumers is playing a role, and an agent only needs to exchange information directly with a subset of all agents in the information network according to the information structure. This information sharing model is combined with the distributed MPC method to achieve power balance. The idea is that the system, as a total, reaches the same balance as if it could bargain with all end-users directly. However, now there is an ordering by information distance to neighbors from whom an end-user buys his power: if the power is available at the direct neighbors, the end-user will buy from this neighbor, and the power coordination is done locally. In the case that an end-user needs to buy from a neighbor that is not a direct neighbor, he must bargain through his neighbor's neighbor connections until an end-user wants to sell.

We apply the distributed MPC method presented in Giselsson et al. [16], [17] to our information sharing model. The distributed method is based on dual-decomposition as presented in Rantzer [52]. The idea is, therefore, to split the computation into smaller sub-problems, that can be computed at the household level. An information structure specifying the exchange information at each time step is introduced. This way, the end-user can make his control decision based on price incentives from virtually connected neighbors, local imbalance information predictions, his own constraints and his own predictions. This approach is expected to scale better than a centralized approach in a large network, as confirmed by Chapters 4-6 of this thesis.

1.2 Contributions and Outline of the Thesis

The main contributions of the thesis are:

- We develop an information sharing model to facilitate distributed control in a multiple consumers and prosumers network. Here, we introduce a virtual information sharing network which is to be distinguished from the physical power grid.

- We give four rules for the design of the information sharing network, and show an example of how to make an information network in the current power grid given the design flexibilities when the four rules are imposed.

- We apply a dual decomposition method to embed distributed generation from μ-CHPs in the power grid.

- We explicitly include on-off constraints, a minimum on (off) time and predictions to the μ-CHP network. This is done in the distributed MPC framework, and challenges due to the non-convex constraints are solved both in a QP setting and an MIQP setting.

- We investigate a more realistic model of the μ-CHP which also considers the heat output. In this case, the dynamics of heat buffers are also included in the problem. We compare four different system setups, and choose the MIQP solver to find the solution to the local optimization problems due to the binary on-off decisions.

- We coordinate demand response for heavy demand consumers using the information sharing model together with a distributed MPC algorithm implemented in a parallel fashion.

The rest of the thesis is organized as follows:

Chapter 2 presents the optimal control preliminaries, and reviews the theory we use to solve our control problems in the smart grid in a completely distributed way.

Chapter 3 is based on our paper Larsen et al. [41] and presents a new information sharing model for a smart grid setting. In the future, global energy balance of a smart grid system can be achieved by its agents deciding on their own power demand and production (locally) and the exchange of these decisions. We develop a network model that describes how the information of power imbalance of individual agents can be exchanged in the system. Our model facilitates a completely distributed method to achieve power balance in the system. Additionally, dynamics, constraints and forecasts of each agent can be conveniently involved.

Chapter 4 is based on our work presented in Larsen et al. [36], [37], and [39]. The aim is to achieve a balance of power in a group of prosumers, based on a price mechanism, i.e. to steer the difference between the total production and consumption of power to zero. We set the information network topology such that the prosumers exchange price (power) information with their neighbors according to a chosen information network topology. Based on the exchanged information and the prosumer's own measured power demand, each prosumer uses a local control strategy to turn on and off its power generator to cooperatively achieve the global balance. First we show the results with no input constraints. Second we include on-off constraints and power modulation. In the second case, we work in the Model Predictive Control framework. More specifically, the local control strategy results from a distributed model predictive control method based on dual decomposition and sub-gradient iterations as in Giselsson et al. [16]. The method achieves a unique dynamic price signal for each prosumer. Simulation results with realistic data validate the method.

Chapter 5 is based on our work presented in Larsen et al. [40] and considers heat and power production from micro μ-CHP systems and heat storage in a network of households. The goal is to balance the local heat demand and supply in combination with balancing the power supply and demand in the network. The on-off decisions of the local generators are done completely distributed based on local information and information exchange with a few neighbors in the network. This is achieved by using an information sharing model with a distributed model predictive control method based on dual-decomposition and sub-gradient iterations Giselsson et al. [16]. Because of the binary nature of the decisions, a mixed integer quadratic problem is solved at each agent. The approach is tested with simulation, using realistic heat and power demand patterns. We conclude that the distributed control approach is suitable for embedding the distributed generation at the household level.

Chapter 6 concerns demand side control in the smart grid, and is based on our paper Larsen et al. [35]. In the future, a global energy balance of a smart grid system can be achieved by its agents deciding on their own power demand locally and the exchange of these decisions. We model a network of households with washing machine programs that can be shifted in time so that the overall power demand is flattened. The network model describes how the information of power imbalance of individual agents can be exchanged in the system. Additionally, dynamics, washing machine constraints and power demand forecasts of each agent are included. Compared to existing network models with hierarchical structures, our developed model, together with a market mechanism, achieves the power balance in the system in a completely distributed way. The market mechanism is a distributed MPC scheme based on dual decomposition and sub-gradient iterations. We provide results with a realistic power and washing machine demand pattern and we test scalability of the problem. Finally, we provide insights in the scalability of the algorithms.

In the end, Chapter 7 concludes the thesis.

1.3 The Flexines Project

This thesis project was carried out as part of the Flexines[2] project, which includes the partners Hanze University of Applied Sciences, University of Groningen, DNV

[2] http://flexines.org/

KEMA, TNO, GasTerra, Icopal, Energy Valley, and RenQi. This three year project finished in April 2012.

The perspective of Flexines is that in a more sustainable The Netherlands the power supply pattern will be more irregular. In a free market, the electricity price rates will fluctuate accordingly. Thus, a power excess means a decrease in prices and a power shortage means an increase in prices. Therefore, Flexines made an Energy Management System (EMS) to help the end-users take advantage of such a situation. This is in the spirit of the project slogan ”People in Power!”. The EMS turns on and off appliances taking into account comfort-level and prices. The EMS needs an input from the network in form of a forecast of the price rates, and it considers current and forecasted energy usage inside the household. The EMS system was tested in the RenQi lab. A test setup including a washing machine, a refrigerator, a μ-CHP system, and solar panels showed that the EMS successfully managed the equipment to lower the electricity bill while staying within the comfort-levels specified by the owner. Figure 1.1 shows the lab setup.

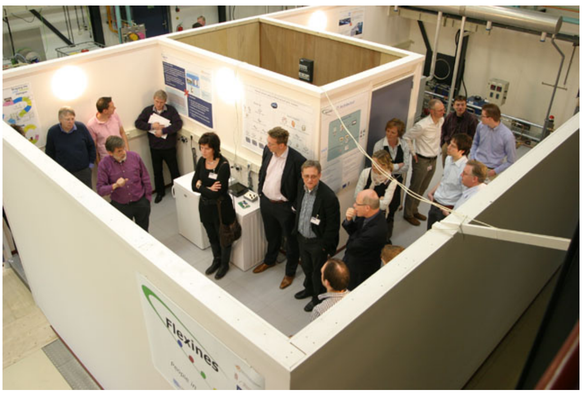


Figure 1.1 – The Flexines EMS test setup at the RenQi lab.

Our role in the project is to consider a network of households with EMS installed. In this case, it is important to have a market mechanism such that the network as a total achieves a balance between supply and demand. Thus, the end-users have to receive the right price signal so that the local decisions are coordinated. We have to consider how a forecast of the price affects the demand and supply patterns in the network, which again affects the forecast of the prices. It is in the spirit of the

Flexines project that the end-user determines the comfort-levels. Given the flexibility in the electric appliances, the appliances are coordinated through distributed interactions in the network.

We wish to find a price mechanism that stimulates a network of consumer-generators to a flat electricity demand (or a balance between supply and demand). In other words, the net consumption should be close to a target level. The user is motivated to participate by a short term goal of a cheaper electricity bill, and an environmentally aware user can be motivated by a more efficient use of the resources. The network gets a reward in terms of facilitating the highest energy flow in the network for the lowest cost, because components like the transformer can operate closer to their maximal capacity. If the net consumption is flat, but lower than the critical load, more users can be connected on one transformer node, resulting in a cheaper network structure. This is a long term goal for the network. We assume this is in the interest of each user since the electricity transport becomes cheaper.

1.4 Ongoing Work

This research serves as the first of a series of new studies on smart grids, and in fact the methods are already being taken to a next level. In a current project between the University of Groningen and DNV KEMA, the information sharing model with distributed MPC is incorporated in the Universal Smart Energy Framework (USEF) framework. This is a framework developed by the Smart Energy Collective, a consortium of companies, and it provides design, specifications, and implementation guidelines for a smart energy system. More information about USEF can be found in [10].

On another note, the methods used for this thesis may be applied to other types of networks as well. In a current project between the University of Groningen, DNV KEMA, Gasunie, Gasterra and Hanze University of Applied Sciences, the method is applied to the smart gas grid. There, the aim is to determine an optimal storage size and use a real-time pricing mechanism such that a owner of a bio-gas generator can support his own local gas demand and possibly a portion of a neighbor's gas demand.

CHAPTER 2

Optimal Control Preliminaries

This chapter reviews the general techniques that we will use to solve our optimal control problems in Chapters 3 - 6. We start by reviewing some properties of convex optimization given in [6]. Then we describe how these ideas can be used for cooperative distributed optimal control of a set of agents with coupled dynamics as presented in [51] and [52]. In the end, we introduce the concept of Model Predictive Control (MPC), see e.g. [8], and we describe the MPC version of the distributed optimal control technique [16], [17]. The technique is based on dual-decomposition and sub-gradient iterations, and is also referred to as a price mechanism.

2.1 Optimization and Duality

This section reviews concepts from convex optimization, in particular duality and the Lagrange dual function. We introduce a few key concepts in the setting of a static optimization problem. This way, we will have the terminology ready for the dynamic situation in Sections 2.2 and 2.3. A more detailed description of the theory reviewed in this section, can be found in [6].

Definition 2.1.1. *[6] A* **convex set** *C is such that the line segment between any two points $x_1, x_2 \in C$ lies in C, i.e., we have $\theta x_1 + (1-\theta)x_2 \in C$ for all $\theta \in [0,1]$.*

Definition 2.1.2. *[6] A* **convex function** *$f : \mathbb{R}^n \to \mathbb{R}$ is such that $\mathbf{dom} f$ is a convex set and for all $x, y \in \mathbf{dom} f$, and θ with $\theta \in [0,1]$, we have $f(\theta x + (1-\theta)y) \leq \theta f(x) + (1-\theta) f(y)$.*

First, we introduce a general static optimization problem. For a given variable $x \in \mathbb{R}^n$, $x = [x_1, \ldots, x_n]^\mathsf{T}$, we associate a cost $V : \mathbb{R}^n \to \mathbb{R}$, inequality constraints

$f_i(x) \leq 0$, and equality constraints $h_j(x) = 0$ where $f_i : \mathbb{R}^n \to \mathbb{R}$, $h_j : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \dots, m$, and $j = 1 \dots, p$. The *primal problem* is given by

$$\begin{aligned} \text{minimize} \quad & V(x) \\ \text{subject to} \quad & f_i(x) \leq 0, \quad i = 1, \dots, m, \\ & h_j(x) = 0, \quad j = 1, \dots, p. \end{aligned} \tag{2.1}$$

We assume that the domain of the primal problem

$$\mathscr{D} = (\cap_{i=0}^{m} \text{dom } f_i) \bigcap (\cap_{j=0}^{p} \text{dom } h_j), \tag{2.2}$$

is non-empty, and denote the optimal value of (2.1) by V^*. However, solving the primal problem implies that we need to explicitly take into account the hard constraints in (2.1), to find the optimal value of x.

A powerful tool for solving problems without the need to explicitly solve the hard constraints is the method of Lagrange multipliers. To make use of this method we define the Lagrange dual function $L(x, \nu, \lambda)$

$$L(x, \nu, \lambda) = V(x) + \sum_{i=1}^{m} \nu_i f_i(x) + \sum_{i=1}^{p} \lambda_i h_i(x), \tag{2.3}$$

where the Lagrangian multiplier vectors $\nu \in \mathbb{R}_+^m$ and $\lambda \in \mathbb{R}^p$ are associated with the inequality and equality constraints in (2.1). This way, the constraints in the primal problem are relaxed. The constraints are no longer hard, but violating the constraints results in an additional cost that is linear in the amount of violation. Taking the minimum of $L(x, \nu, \lambda)$, for fixed ν and λ over $x \in \mathscr{D}$ provides a lower bound for the optimal value of V^*. To verify this, we observe that by definition, for any feasible point $\tilde{x} \in \mathscr{D}$ of (2.1), we have

$$\inf_{x \in \mathscr{D}} L(x, \nu, \lambda) \leq L(\tilde{x}, \nu, \lambda), \tag{2.4}$$

for any ν, λ. Further, for any feasible point $\tilde{x}$, we have

$$L(\tilde{x}, \nu, \lambda) \leq V(\tilde{x}), \tag{2.5}$$

because the terms $\sum_{i=1}^{m} \nu_i f_i(\tilde{x})$ and $\sum_{i=1}^{p} \lambda_i h_i(\tilde{x})$ in (2.3), are negative and equal to zero respectively at $\tilde{x}$. Finding the best lower bound is called the *dual problem* and is given by

$$L^* = \sup_{\nu \geq 0, \lambda} \inf_{x \in \mathscr{D}} L(x, \nu, \lambda), \tag{2.6}$$

where $\geq$ here means component-wise greater or equal.

Remark 2.1.1. *Notice that ν_i in (2.6) has to be positive because of the inequality constraints in (2.1). When $f_i(x)$ is negative, and the constraint is satisfied, the term $\sum_{i=1}^{m} \nu_i f_i(x)$ in (2.3) contributes to lower the value of $L(x, \nu, \lambda)$. While, if the constraint is violated $f_i(x) > 0$, the term acts as a penalization. On the other hand, $h_i(x)$ should be penalized both if the value is positive and negative. Therefore, λ_i can be both positive and negative.*

We denote the optimal value of the dual problem by L^*. Similarly to (2.6), the optimal value of the primal problem can also be written in terms of the Lagrangian dual function

$$V^* = \inf_{x \in \mathscr{D}} \sup_{\nu \geq 0, \lambda} L(x, \nu, \lambda), \tag{2.7}$$

since $\sup_{\nu \geq 0, \lambda} L(x, \nu, \lambda) = V(x)$ if x is feasible and $\sup_{\nu \geq 0, \lambda} L(x, \nu, \lambda) = \infty$ if x is not feasible. With definition (2.7) we are ready to define the weak duality property.

Proposition 2.1.1. *[6]* **Weak duality***: The optimal value of the Lagrangian dual problem is always smaller or equal to the optimal value of the primal problem, i.e., $L^* \leq V^*$.*

Weak duality, given in Proposition 2.1.1, always holds, even if the primal problem is not convex. Sometimes, however, we use the concept of duality gap instead of weak duality.

Definition 2.1.3. *[6] The* **optimal dual gap** *is defined to be $V^* - L^*$.*

It follows from the weak duality property that the dual gap is always nonnegative. Furthermore, when the bound in Proposition 2.1.1 holds with equality, the optimal dual gap is zero, and then we have strong duality.

Definition 2.1.4. *[6]* **Strong duality** *holds if and only if $V^* = L^*$.*

There exist conditions to ensure that strong duality holds, but usually we have strong duality if the cost and inequality constraints are convex, and the equality constraints are linear. In particular, Slater's theorem [6] guarantees that strong duality holds if in addition the inequality constraints are replaced with strict inequalities $f_i(x) < 0$ for $i = 1, \ldots, m$.

If strong duality holds it follows from (2.6) and (2.7) that

$$\inf_{x} \sup_{\nu \geq 0, \lambda} L(x, \nu, \lambda) = \sup_{\nu \geq 0, \lambda} \inf_{x} L(x, \nu, \lambda). \tag{2.8}$$

This property is called the *strong min-max property* or *saddle-point property*.

2.1.1 Sub-gradient Method

Convex optimization problems can be solved by sub-gradient methods, see e.g. [4] and [55]. This is an iterative method, that is often used with decomposition methods in large scale systems to solve the problem in a distributed manner.

Suppose we want to solve

$$\underset{x\in\mathbb{R}^n}{\text{minimize}} \quad V_0(x), \tag{2.9}$$

where $V_0 : \mathbb{R}^n \to \mathbb{R}$ is a convex function. To obtain an iterative solution we need to find a sequence $\{x(k)\}_{k=0}^{k=\infty}$ that converges to the optimal vector x^* in some sense. The idea is that we choose a direction $g(k)$ and make a step in this direction with some step-size coefficient $\gamma(k)$, such that $V_0(x(k+1)) < V_0(x(k))$.

For implementation of a iterative method, we stop the calculations at some iteration k and we accept $x(k)$ as a sufficiently good approximation for a solution. The question is now, how do we determine what direction and step-size to choose?

Definition 2.1.5. *[55]* **Sub-Gradient:** *Let $f : \mathbb{R}^n \to \mathbb{R}$ be a convex function. We say that a vector $g \in \mathbb{R}^n$ is a sub-gradient of f at a point $x \in \mathbb{R}^n$ if $f(y) \geq f(x) + g^\mathsf{T}(y-x)$, $\forall y \in \mathbb{R}^n$.*

Definition 2.1.6. *[55]* **Subdifferential:** *The set of all sub-gradients of f at x is called the subdifferential $\partial f(x)$.*

In the sub-gradient method, as the name suggests, we calculate a sub-gradient $g(k) \in \partial V_0(x(k))$ at each iteration k and make a step in the opposite direction. The iterative formula is therefore

$$x(k+1) = x(k) - \gamma(k)\tau(k)g(k) \tag{2.10}$$

where $\gamma(k) > 0$ is a step size, and $\tau(k) > 0$ is a scaling coefficient included to ensure that the norm of the vector $\tau(k)g(k)$ is bounded. One choice for this coefficient could be $\tau(k) = \frac{1}{||g(k)||}$. The convergence depends on the coefficient $\gamma(k)\tau(k)$, for more details see e.g. [55].

The sub-gradient method can also be applied to the dual of (2.1). Given a point $x(k)$, we can find the optimal ν and λ for this point by the means of the method. This can be done because the Lagrangian dual function (2.3) is concave in ν and λ. In fact, since the dual function is the point-wise infimum of a family of affine functions of ν, λ, it is concave even when the problem (2.1) is not convex [6]. We define

$$L_{D,k}(\nu(k),\lambda(k)) = \inf_{x(k)} L(x(k),\nu(k),\lambda(k)), \tag{2.11}$$

for iteration k. The iterative updates of the Lagrangian multipliers are given by

$$\nu(k+1) = \max(0, \nu(k) + \gamma(k)\tau(k)f(x(k))), \tag{2.12}$$

$$\lambda(k+1) = \lambda(k) + \gamma(k)\tau(k)h(x(k)), \tag{2.13}$$

where $\begin{bmatrix} f(x(k)) \\ h(x(k)) \end{bmatrix}$ is a sub-gradient of $L_{D,k}(\nu(k),\lambda(k))$ at the current point $x(k)$ of the iterations, $\gamma(k) > 0$ is again a step size, and $\tau(k) >$ is a scaling coefficient included to ensure that the norms of the vectors $\tau(k)f(x(k)), \tau(k)h(x(k))$ are bounded. When $x \in X_0$ and X_0 is a compact set, the sub-gradients are uniformly bounded and we can use $\tau(k) = 1$, see [55].

Finally, notice that when the objective function is differentiable the search direction of the sub-gradient iteration is the same as that of gradient descent. Therefore, to find the local minimum of the differentiable function, we take steps proportional to the negative of the gradient of the function in the current point $x(k)$.

Remark 2.1.2. *Often merely a subset of the constraints of the original problem are included in the Lagrangian, and the remaining constraints are treated explicitly in the problem. We will see in Section 2.2, that we indeed only include complicating constraints in the Lagrangian.*

2.1.2 Price Interpretation of the Lagrangian Multipliers

The Lagrangian multipliers in the dual problem are often interpreted as prices or shadow prices. This interpretation comes from economics and game theory literature [6]. In [6], a variation of the following example is given.

Suppose x in (2.1) represents how a firm operates, and $V(x)$ is the cost ($-V(x)$ is the profit) for operating at this point. The inequality constraint $f_1(x) \leq 0$ represents a constraint on the amount of available materials. Further, we assume that all other constraints are zero.

The firm is allowed to pay a price ν_1 per unit violation of the material constraint. Thus, the firm has an additional cost $\nu_1 f_1(x)$. The optimal duality gap is the smallest possible benefit for the firm to be allowed to pay to violate the constraints, and hence buy extra material. Notice that if the constraint is not tight the firm earns money since $\nu_1 \geq 0$.

Definition 2.1.7. *[6]* **Shadow prices** *are the set of dual optimals that are attained when strong duality holds.*

When strong duality holds, the dual optimal, ν^* in this example, is called the equilibrium price or shadow price, see Definition 2.1.7. In this case, there is no advantage for the firm to pay to violate the constraints. Therefore, if a resource i is priced less than ν_i^* the firm should buy, because it would increase the profit more than it costs for the resource. And similarly if the price is higher than ν_i^*, the firm should sell, since the profit for selling is higher than the drop in profit.

In a network with multiple decision makers, it is useful to work with price like concepts when dealing with allocation problems [57]. An iterative adjustment of the price, as in Eq. (2.13), is just like a market equilibrium process where the price is adjusted to for example match supply and demand.

2.2 Optimal Control in a Network of Decision Makers

We consider a global optimal control problem over a network consisting of n agents. Agent i will be dynamically connected to agent j by a weight A_{ij}, and it is therefore useful to think of the network in terms of a graph. Thus, we introduce a weighted directed graph (U_n, E_n) with n agents, where $U_n = \{1, \ldots, n\}$ is a finite nonempty agent set, and $E_n \subseteq U_n \times U_n$ is an edge set. There is an edge in the graph $(i, j) \in E_n$ whenever agent i and agent j are connected. Therefore

$$A_{ij} \neq 0 \quad \text{if and only if} \quad (i, j) \in E_n, \tag{2.14}$$

where self-loops are allowed, i.e., $A_{ii} \neq 0$. Figure 2.1 displays a graph where $n = 5$, and the arrows represent the edges. In this example, agent 1 has two neighbors, namely agent 2 and agent 3.

In game theoretic terms, we are considering a network of decision makers that are dynamically coupled, but have access to different information about the underlying uncertainties. Optimization of such a problem is called team-optimization. The aim is to reformulate the problem so that is splits into several sub-problems, one for each agent. The solution of the distributed solutions should then be combined into the solution of the original global problem. By introducing prices, the

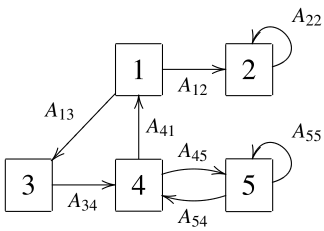


Figure 2.1 – A graph with five agents. The arrow from agent i to agent j indicate the connections with weights A_{ij}.

problem is reformulated from a team-minimization problem to a non-cooperative game with additional players [51]. First, we review the Linear Quadratic Regulator, then we see how this solution relates to the solution of the reformulated problem.

2.2.1 The Linear Quadratic Regulator Problem in a Network

Good references on the discrete Linear Quadratic Regulator (LQR) are [34] and [42]. Here we summarize the main results for a stochastic formulation of the LQR problem for networks, because in Section 2.2.2 we will use this result to establish the relationship between the optimal solution found by the LQR method and the optimal solution found by dual-decomposition methods.

We consider a discrete time network with n agents, where at time k agent i has a state $x_i(k) \in \mathbb{R}$, a control input $u_i(k) \in \mathbb{R}$, and a disturbance $w_i(k) \in \mathbb{R}$ from a set of independent and identically distributed Gaussian random variables with mean $\mu = 0$ and a given variance σ^2. The agents are dynamically coupled though state equations

$$x_i(k+1) = \sum_{j=1}^{n} A_{ij} x_j(k) + B_{ii} u_i(k) + w_i(k),\ i = 1, \ldots, n, \tag{2.15}$$

where A_{ij} weights the connections in the network, and the input weight B_{ii} equals one. This means that each agent can only control one input signal $u_i(k)$ directly.

In order to write the system in a compact form we define the vectors

$$x(k) = [x_1(k) \dots x_n(k)]^\intercal,$$
$$u(k) = [u_1(k) \dots u_n(k)]^\intercal,$$
$$w(k) = [w_1(k) \dots w_n(k)]^\intercal.$$

Then, the compact form of the distributed system is given by state equation

$$x(k+1) = Ax(k) + Bu(k) + w(k), \tag{2.16}$$

where system matrix $A \in \mathbb{R}^{n\times n}$ is stable and consists of the components A_{ij} in (2.15), and the input matrix B equals the identity matrix.

To measure the performance of the system, we define a quadratic objective function

$$V_u(K) = \frac{1}{K}\sum_{k=1}^{K} \mathbf{E}[|x(k)|_Q^2 + |u(k)|_R^2], \tag{2.17}$$

where $Q \in \mathbb{R}^{n\times n}, R \in \mathbb{R}^{n\times n}$ are positive semidefinite weight matrices, and we use the notation $|x(k)|_Q^2 = x^T(k)Qx(k)$. The first term of (2.17) penalizes the deviation of $x(k)$ from zero, and the second term penalizes the use of the control input $u(k)$. In addition, we assume that both Q and R are diagonal, so that the objective function is separable. We will see why this property is useful in Section 2.2.2.

The goal of the LQR problem is to find the control input that minimizes the infinite horizon cost based on (2.17). The optimal value of this problem is therefore given by

$$\begin{aligned} \underset{u}{\text{minimize}} \quad & \limsup_{K\to\infty} V_u(K), \\ \text{subject to} \quad & (2.16). \end{aligned} \tag{2.18}$$

In [7] it is stated that "Standard assumptions are that (Q,A) is observable, (A,B) is controllable, and R is positive definite $R > 0$". With these assumptions the controller that attains the optimal value of (2.18) is in fact a constant state-feedback

$$u(k) = -Lx(k) \tag{2.19}$$

where the feedback matrix $L \in \mathbb{R}^{n\times n}$ can be found with standard LQ control theory in the following way. Suppose $P > 0$ is the unique stabilizing solution to the discrete time algebraic Riccati equation

$$A^\intercal PA - P - A^\intercal PB(B^\intercal PB + R)^{-1}B^\intercal PA + Q = 0, \tag{2.20}$$

then L in (2.19) is given by

$$L = (R + B^{\mathsf{T}}PB)^{-1}B^{\mathsf{T}}PA. \tag{2.21}$$

In general, L is therefore a full matrix. As a consequence of this, agent i needs information from all other agents in the network to update its state, since substituting (2.19) into (2.16) yields

$$x(k+1) = (A - BL)x(k) + w(k). \tag{2.22}$$

This need for full state information is however not desirable when we have a distributed control strategy in mind. In [52], it is suggested that one can use an approximation of L, by only considering the diagonal elements $L_D = diag(L)$ of the feedback matrix. By comparing the performance (2.17) using the optimal L and the approximation L_D, we can determine how sub-optimal this approach is on a case by case basis. Drawbacks of this method are that one needs full system information while designing the controller, and changes in the system cannot be incorporated. When there are communication limitations in the graph, i.e., if each agent in Fig. 2.1 can only get information from connected neighbors, the L matrix might be computed in a distributed manner. It has been shown in [43] that an approximation of the feedback matrix L also can be obtained with completely distributed gradient iterations. We call the approximated feedback matrix $\bar{L}$. Then the local suboptimal feedback controllers $\bar{L}_i$ are updated for the distributed systems using information from neighbor connections only. Moreover, $\bar{L}$ inherits the sparsity structure of A. Thus, we can compute a control $\bar{u}(k) = \bar{L}x(k)$ only having information from the connected neighbor agents in a graph such as Figure 2.1.

We are, however, interested in distributing the control problem itself. This means that we will distribute the computational effort to solve (2.18) over the agents. By using a price mechanism, based on dual-decomposition and sub-gradient iterations, to determine the control inputs, we can achieve such a distributed algorithm. Therefore, we will first see how the dual problem presented in Section 2.1 can be used to decompose the problem in a way that can be used for distributed control of agents in a network.

2.2.2 Dynamic Dual-Decomposition

The general dual-decomposition idea was first presented in the 1960s by Everett [15], but related ideas were presented earlier. Later, decomposition was also applied for dynamical systems as well as in large-scale optimization problems due to the possibility of distributed implementation. The survey [47] is a nice reference on dual methods and decomposition.

This section reviews dynamic dual-decomposition for distributed control as presented in [52]. In [52] it has been shown that dynamic price mechanisms can be used for decomposition and distributed optimization of feedback systems. Moreover, a relation between the specific choice of decomposition and the LQR solution is given. We start from the network of agents with coupled dynamics as presented in Section 2.2.1, where the system is driven by white noise, i.e. $w_1(k),\ldots,w_n(k)$ are independent white noise processes. Then, we use Lagrangian relaxation of some of the constraints in order to decompose the problem.

In state equation (2.16), we see that the right hand side depends on neighbor states through the terms $\sum_{j\neq i} A_{ij}x_j$. We say that we have a coupled system, because the update of one agent also depends on information from neighbors. To decouple the state equation (2.16), each agent introduces a local variable, $v_i(k) \in \mathbb{R}$, representing the expected influence from other agents on its state equation [9], [52]. Thus, the state equation is now given by the fully decoupled state equations

$$x_i(k+1) = A_{ii}x_i(k) + v_i(k) + B_{ii}u_i(k) + w_i(k),\ i = 1,\ldots,n, \tag{2.23}$$

with the additional equality constraints

$$v_i(k) = \sum_{j\neq i} A_{ij}x_j,\ i = 1,\ldots,n, \tag{2.24}$$

since the expected influence from neighbors ideally should agree with the real influence from neighbors. Equations (2.23) and (2.24) in compact form yields

$$\begin{aligned} x(k+1) &= (A_D)x(k) + Bu(k) + v(k) + w(k) && (2.25)\\ v(k) &= A_o x(k) && (2.26) \end{aligned}$$

where we have defined $A_D = \mathrm{diag}(A)$ and $A_o = A - A_D$.

To decompose the control problem (2.18), we use Lagrangian decomposition to include the constraint (2.26) in the objective. Thus, notice that we here only include a subset of the constraints in (2.18) in the Lagrangian. We introduce the Lagrangian multiplier vector $\lambda(k) \in \mathbb{R}^n$, and write the problem as a max min problem

$$\begin{aligned} \underset{\lambda}{\text{maximize}} \quad \underset{u,v}{\text{minimize}} \quad & \limsup_{K\to\infty} V_{u,v,\lambda}(K),\\ \text{subject to} \quad & (2.25), \end{aligned} \tag{2.27}$$

as explained in Section 2.1, where we define

$$V_{u,v,\lambda}(K) = \frac{1}{K}\sum_{k=1}^{K}\mathbf{E}[|x(k)|_Q^2 + |u(k)|_R^2 + \lambda^{\intercal}(k)(v(k) - A_o x_j(k))]. \tag{2.28}$$

The multipliers $\lambda(k)$ are also called shadow prices, see Section 2.1.2. To obtain a set of decoupled minimization problems, we interchange the minimization and the sum over agents in (2.27), and we rearrange the terms such that the problem is now given by

$$\begin{aligned} \underset{\lambda}{\text{maximize}} \quad \sum_{i=1}^{n} \quad & \underset{u_i,v_i}{\text{minimize}} \quad \limsup_{K\to\infty} V_{u_i,v_i,\lambda}(K), \\ & \text{subject to} \quad (2.23), \end{aligned} \tag{2.29}$$

where we define

$$V_{u_i,v_i,\lambda}(K) = \frac{1}{K}\sum_{k=1}^{K}\mathbf{E}[\overbrace{|x_i(k)|_{Q_i}^2 + |u_i(k)|_{R_i}^2}^{\text{local}} + \overbrace{\lambda_i^{\intercal}(k)v_i(k)}^{\text{charged}} - \overbrace{(\sum_{j\neq i}\lambda_j^{\intercal}(k)A_{ji})x_i(k)}^{\text{received}}]. \tag{2.30}$$

This is possible since R, Q in (2.29) are assumed to be diagonal. We observe that the minimizations over x_i and v_i in (2.29) are now completely decoupled, given the shadow prices $\lambda(k)$. Thus, the problem is reformulated from a team-optimization problem to a non-cooperative game with additional players [51]. The new players are market makers associated with state variables shared by agents through the A matrix in state equation (2.16). They adjust the prices to take advantage of violation of constraints (2.26). The agents can now pay each other to modify $x_i(k)$ and find a common equilibrium.

However, central coordination is still needed to obtain the prices. In Section 2.3, we include sub-gradient iterations in order to make the method completely distributed.

Remark 2.2.1. *In [52] a special interpretation of the terms of the objective function in (2.29) is given. The first two terms represent the agent's local cost, the third term represents what the agent expects others to charge him, and the fourth term represents what the agent receives from other agents.*

Now we see how the solution of (2.29) relates to the solution of the LQR problem (2.18). In fact, the solutions are equivalent, see Theorem 2.2.1 and Theorem

2.2.2. For Theorem 2.2.1, we define V^* to be the optimal value of (2.18) and we denote the objective in (2.29) by $V_i(x_i, u_i, v_i, \lambda)$.

Theorem 2.2.1. *[52] Consider control laws $\bar{u}(k) = -\sum_j L_{ij}\bar{x}_j(k)$ and corresponding stationary solutions to the state equations (2.15). For given white noise processes $w_i(k)$, suppose there exist price processes $\lambda_i(k)$ such that*

$$V_i(\bar{x}_i(k), \bar{u}_i(k), \sum_{j\neq i} A_{ij}\bar{x}_j(k), \lambda(k)) \leq \alpha \min_{\mu_i, v_i} V_i(x_i(k), u_i(k), v_i(k), \lambda(k)), \qquad (2.31)$$

when minimizing over control laws $u_i(k) = \mu_i(x(k))$, $v_i(k) = \eta_i(x(k))$ and stationary solutions of (2.23) and (2.24). Then

$$V^* \leq \limsup_{K\to\infty} \frac{1}{K} \sum_{k=1}^{K} \mathbf{E} \sum_{i=1}^{n} [|\bar{x}_i(k)|^2_{Q_i} + |\bar{u}_i(k)|^2_{R_i}] \leq \alpha V^*. \qquad (2.32)$$

Theorem 2.2.2. *E.g. [34] Suppose we have (2.25), (2.26) and $P > 0$, and that L, M is determined by*

$$|x(k)|^2_P = \min_{u(k)} (|Ax(k) + Bu(k)|^2_P + |x(k)|^2_Q + |u(k)|^2_R), \qquad (2.33)$$

$$L = (R + B^T PB)^{-1} B^T PA, \qquad (2.34)$$

$$M = P(A - BL). \qquad (2.35)$$

Given the white noise $w(k)$, let $\bar{x}(k), \bar{u}(k)$ and $\lambda(k)$ be defined by

$$\bar{x}(k+1) = A\bar{x}(k) + B\bar{u}(k) + w(k), \qquad (2.36)$$

$$\bar{u}(k) = -L\bar{x}(k), \qquad (2.37)$$

$$\lambda(k) = -M\bar{x}(k). \qquad (2.38)$$

Then (2.32) holds with $\alpha = 1$ for $i = 1, \dots n$.

The proofs involve a standard application of duality theory and LQ optimal control theory, and are given in e.g. [34].

2.3 Model Predictive Control (MPC)

In our applications we aim to control power producing (consuming) devices in a network. These devices are subject to several operational constraints, which means that the control inputs $u_i(k)$ take their values in a constrained set $U \subseteq \mathbb{R}^n$. As a consequence, an infinite horizon optimal control problem, as described in Section 2.2 cannot be solved by an algebraic Riccati equation, because the new problem is subject to hard constraints over the inputs and states. In general we cannot even find a closed loop expression $u(k) = f(x(k))$ that solves the infinite horizon control problem. We therefore choose to solve the optimal control problem in the MPC setting. In the MPC setting, a modified optimal control problem is solved at each time-step k over a *finite horizon* K_{pred}. At each time-step k new measurements, predictions of future states, and disturbances can be taken into account. This idea is also known as the *receding horizon principle*, and is implemented for MPC [8]. MPC is known to be robust with respect to external disturbances, and it is a systematic approach to take both static and dynamic constraints into account [46]. The solutions are sub-optimal compared to solving the infinite horizon problem, but we ensure that the constraints can be met for all agents i for all time k.

The receding horizon implementation of the open-loop optimal control solution subject to constraints at each sample time (MPC) was a new concept in the 1970s [53]. Today, the method is widely used in process industry [8], and the applications to power network related topics are many, see e.g. [24], [64].

MPC can be viewed as a methodology that is extending limitations of classical optimal control. Here we are interested in MPC because it is a systematic way to include input and state constraints. However the methodology can also handle structural changes, large time-delays, non-minimum phase and unstable systems. The technique only requires a model of the system, a model of the disturbances, a measurement of the current states, and an objective function for the minimization over the finite horizon.

Here we define a MPC problem corresponding to (2.18), but now including input and state constraints. We review a distributed MPC method that finds the control inputs $u_i(k)$, for $i = 1, \dots, n$, in a completely distributed manner. First we introduce the centralized MPC, and then we continue with the distributed MPC, in which the central MPC problem is decomposed into substantially smaller sub-problems. We use the same dual-decomposition technique as reviewed in Section 2.2.2, but now we also include sub-gradient iterations. Thereby, each sub-problem is iteratively solved independent and combined into a global solution.

2.3.1 Centralized MPC in a Network of Decision Makers

We introduce a new time variable $\tau = k,\dots,k+K_{\text{pred}}$, in order to label the predictions over the prediction horizon K_{pred}. Further, we use hat notation to indicate predictions. For example, the sequences $\{\hat{x}_i(\tau)\}_{\tau=k}^{k+K_{\text{pred}}}, \{\hat{w}_i(\tau)\}_{\tau=k}^{k+K_{\text{pred}}}$ are predictions for state $x_i(k)$ and disturbance $w_i(k)$ over the prediction horizon. The predictions of the state $\hat{x}_i(\tau)$ are based on (2.16).

In line with the literature on MPC, we do not use expectation values as in the stochastic setting in Section 2.2. Therefore, the disturbance $w_i(k)$ is treated as an external signal in the MPC setting instead of a stochastic variable. As a consequence, the formulation needs to provide estimates for $\hat{w}_i(\tau)$ over the prediction horizon. Further, the true value of $w_i(k)$ is measured at each time-step k and included in the problem.

In the *centralized MPC* problem, the goal is to find the optimal control sequence $\{\hat{u}(\tau)\}_{\tau=k}^{k+K_{\text{pred}}}$ that minimizes the objective function over the horizon $\tau = k,\dots,k+K_{\text{pred}}$, given prediction models, initial conditions and convex constraints. Instead of (2.17), we define the MPC objective function to be

$$V_{i,u}(k) = |x_i(k)|^2_{Q_i} + |u_i(k)|^2_{R_i}, \tag{2.39}$$

and

$$\begin{aligned} V_u(k) &= |x(k)|^2_Q + |u(k)|^2_R, \\ &= \sum_{k=1}^{n} V_{i,u}(k). \end{aligned} \tag{2.40}$$

Thus, the problem is to solve

$$\begin{aligned} \underset{\hat{u},\hat{v}}{\text{minimize}} \quad & \sum_{\tau=k}^{k+K_{\text{pred}}} V_u(\tau), \\ \text{subject to} \quad & \hat{x}(\tau+1) = A\hat{x}(\tau) + B\hat{u}(\tau) + \hat{w}(\tau), \quad \tau = k,\dots k+K_{\text{pred}}-1 \\ & \hat{x}(\tau)\mid_{\tau=k} = x(k), \\ & \hat{x}(\tau) \in X,\ \hat{u}(\tau) \in U, \qquad\qquad \tau = k,\dots k+K_{\text{pred}}, \end{aligned} \tag{2.41}$$

where the $\hat{w}(\tau)$ is an external signal, constant sets $X,U \in \mathbb{R}^n$ are convex, A,B are defined in Section 2.2, and $V_u(k)$ is defined in (2.40). The minimization is performed at each time step k taking new measurements of $x(k)$ and $w(k)$ into account. Once a finite optimal control sequence $\{\hat{u}(\tau)\}_{\tau=k}^{k+K_{\text{pred}}}$ is obtained, we only implement the first input $u(k) = \hat{u}(\tau)_{\tau=k}$ to system (2.16). The horizon is then shifted one sample $\tau = k+1,\ldots,K_{\text{pred}}+k+1$. Different models for the change in demand $\hat{w}_i(k)$ can be included. This can be a forecast based on information from the agent or on historical data, or in the simplest case; the demand stays the same over the horizon.

Notice that solving problem (2.41) involves the notion of future states $\hat{x}_j(\tau)$ of connected neighbors which again depend on their neighbor connections. Therefore, the solution of problem (2.41) requires that all agents i can access (make assumptions about) the evolution of all states $\hat{x}_j(\tau)$, $j = 1,\ldots,n$ in the network.

Solving the central problem is time consuming for a large network. This is in particular the case with the presence of non-convex constraints, because a difficult combinatorial optimization problem needs to be solved. Therefore, we aim at splitting the problem into smaller sub-problems that can be distributed over the network. Every agent in the network then makes a decision only based on local information. Thus, the central control problem is solved in a distributed manner. In Section 2.3.2, we review the method based on dual-decomposition and sub-gradient iterations, that enables us to solve the problem locally only with information from connected neighbors.

2.3.2 Distributed MPC in a Network of Decision Makers

As in Section 2.2, the idea is to replace the original problem by a set of substantially smaller sub-problems. Then solve each sub-problem in isolation except through a small interface depending on the structure of the original problem. In Section 2.2, we saw that dynamic price mechanisms result from the dual-decomposition method for distributed optimization of feedback systems. In [16], [17], the method is combined with MPC. Models with disturbances $w_i(k)$ that are assumed bounded $|w_i(k)| \leq w_{max}$ and dual-decomposition is presented in [18]. A review of the dual-decomposition technique for MPC with sub-gradient iterations follows in three steps.

The first step to solve the optimal control problem in a completely distributed manner, is to decouple the dynamic constraints in (2.41). As in Section 2.2.2, we introduce a set of variables $\hat{v}_i(\tau) \in \mathbb{R}$. Define

$$\hat{v}_i(\tau) = \sum_{j\neq i} A_{ij}\hat{x}_j(\tau), \tag{2.42}$$

where $\tau = k,\ldots,k+K_{\text{pred}}$, $i = 1,\ldots,n$. Here $\hat{v}_i(\tau)$ is the influence agent i expects to receive from its neighboring agents. The prediction model in (2.41) then yields *decoupled prediction models*

$$\hat{x}_i(\tau+1) = A_{ii}\hat{x}_i(\tau) + \hat{v}_i(\tau) + B_{ii}\hat{u}_i(\tau) + \hat{w}_i(\tau), \tag{2.43}$$

for $\tau = k, \ldots, k+K_{\text{pred}}$, $i = 1, \ldots, n$, additional constraints (2.42), and the change in demands $\hat{w}_i(\tau)$ are assumed bounded $|\hat{w}_i(\tau)| \leq w_{\max}$. The expected influence from neighbors $\hat{v}_i(\tau)$ will be calculated in the local optimization problem of agent i, instead of received directly from the neighbors.

The second step is to use the decomposed model (2.43) for predictions in the optimization problem, while the set of additional constraints (2.42) is included in the MPC dual cost-function. This is the dual-decomposition. The dual decomposition technique (or Lagrange relaxation) requires the introduction of new optimization variables to the problem; the Lagrange multipliers

$$\hat{\lambda}(\tau) = [\hat{\lambda}_1(\tau), \ldots, \hat{\lambda}_n(\tau)]^\intercal \in \mathbb{R}^n \tag{2.44}$$

where $\tau = k, \ldots, k+K_{\text{pred}}$. This variable can be *interpreted as price* signals between neighboring agents [51]. The MPC dual cost-function is given by

$$V_{\hat{u},\hat{v},\hat{\lambda}}(K_{\text{pred}}, k) = \sum_{\tau=k}^{k+K_{\text{pred}}} [V_{\hat{u}}(\tau) + \hat{\lambda}^\intercal(\tau) \cdot \Big(\hat{v}(\tau) - A_o\hat{x}(\tau)\Big)], \tag{2.45}$$

where $V_{\hat{u}}(\cdot)$ is the original MPC cost defined in (2.40), and A_o contains the off-diagonal elements of the information matrix, i.e. $A_o = A - \text{diag}(A)$.

We define initial conditions corresponding to new measurements of state $x_i(k)$ and disturbance $w_i(k)$ for all agents $i = 1, \ldots, n$. In addition, we define range constraints on the state, disturbance, and input of all agents $i = 1, \ldots, n$ over the horizon $\tau = k, \ldots, k+K_{\text{pred}}$. These initial conditions and constraints are given by

$$\begin{aligned} \hat{x}_i(\tau)_{\tau=k} &= x_i(k), \\ \hat{x}_i(\tau) \in X_i, \ \hat{u}_i(\tau) &\in U_i, \end{aligned} \tag{2.46}$$

where X_i, U_i are convex sets.

By minimizing the MPC dual cost over $\hat{u}(\tau)$ and maximizing over $\hat{\lambda}(\tau)$ we will obtain the same value as in (2.41) if the dual gap is zero, recall Definition 2.1.3.

The duality gap is in general zero under convexity assumption on V_u, and linear constraints [16],[6]. The dual problem is given by

$$\begin{aligned} \underset{\hat{\lambda}}{\text{maximize}} \quad & \underset{\hat{u},\hat{v}}{\text{minimize}} \quad V_{\hat{u},\hat{v},\hat{\lambda}}(K_{\text{pred}},k), \\ & \text{subject to} \quad (2.43) \text{ and } (2.46). \end{aligned} \tag{2.47}$$

To obtain a set of decoupled minimization problems, we can use that in (2.47) the sum over the agents $i,\dots,n$ and the minimization over $\hat{u}(\tau)$ can be interchanged. Define

$$V_{\text{agent}}^{i}(K_{\text{pred}},k) = \sum_{\tau=k}^{k+K_{\text{pred}}} \Big(V_{i,\hat{u}}(\tau) + \hat{\lambda}_i(\tau)\hat{v}_i(\tau) - \sum_{j\neq i} \hat{\lambda}_j(\tau) A_{ji}\hat{x}_i(\tau) \Big), \tag{2.48}$$

where $V_{i,\hat{u}}(\cdot)$ is defined in (2.39). This leads to the set of decoupled minimization problems

$$\begin{aligned} \underset{\hat{u}_i,\hat{v}_i}{\text{minimize}} \quad & V_{\text{agent}}^{i}(K_{\text{pred}},k), \\ \text{subject to} \quad & (2.43) \text{ and } (2.46) \end{aligned} \tag{2.49}$$

and (2.47) is reformulated as

$$\begin{aligned} \underset{\hat{\lambda}}{\text{maximize}} \quad \sum_{i=1}^{n} \quad & \underset{\hat{u}_i,\hat{v}_i}{\text{minimize}} \quad V_{\text{agent}}^{i}(K_{\text{pred}},k), \\ & \text{subject to} \quad (2.43) \text{ and } (2.46) \end{aligned} \tag{2.50}$$

Problem (2.50) is still centralized, because global coordination is needed to find the prices. However, only price informations $\hat{\lambda}_j(\tau)$ from connected agents are needed to solve the decoupled minimization problems (2.49). This is the key observation to perform the final step towards a distributed algorithm.

The third and final step to make the problem fully distributed, is to include sub-gradient iterations, see e.g. [16]. This can be done since (2.45) is concave in $\hat{\lambda}(\tau)$, even if the original problem (2.41) is not convex, see e.g. [6]. By including the sub-gradient iterations, (2.50) is approximated. For all $\tau = k,\dots,k+K_{\text{pred}}$ the sub-gradient iterations of the prices are updated according to

$$\hat{\lambda}_{i,r+1}(\tau) = \hat{\lambda}_{i,r}(\tau) + \gamma_{i,r}[\hat{v}_{i,r}(\tau) - \sum_{j\neq i} A_{ij}\hat{x}_{j,r}(\tau)], \tag{2.51}$$

where r labels the sub-gradient iteration, and $\gamma_{i,r}$ is the gradient step size. In this way, the price updates are also distributed, only depending on information from neighboring agents, and the gradient-steps $\gamma_{i,r}$ are chosen such that we converge to the optimum.

Thus, a *distributed MPC* algorithm is obtained. The original information structure is preserved, and as a bonus $\hat{\lambda}(k)$ can be interpreted as a price reference [6] [51]. By reformulating the centralized MPC problem to a distributed MPC problem, prices are introduced to coordinate the decisions in the network. It is a property of the method itself that both the decisions and the dynamic prices are determined iteratively. In this way, we achieve a mutual dependence of decisions and prices. Notice that in the centralized formulation only imbalance is communicated between the agents, but the controller has to have access to all information for all the agents to make the decisions. In the distributed formulation, both the imbalance and the price are communicated between neighboring agents, and there is a local controller present at each agent making decisions only depending on local information. In this section, we explicitly write down the algorithm that is implemented.

2.3.3 Distributed MPC Algorithm

In the algorithm below, the distributed MPC method presented in Section 2.3.2 is sketched. We run the simulation for $k = 0, \ldots, K_{sim}$ number of time-steps. At time-step k, first, each agent $i = 1, \ldots, n$ solves the local control problem (2.49) subject to prediction models (2.43), and constraints (2.46) for all $\tau = k, \ldots, k + K_{\text{pred}}$. The solution of (2.49) is found using a quadratic program solver. Second, the price sequence is updated according to (2.51).

Ideally these steps are repeated till $\varepsilon = 0$ in the relations

$$\hat{v}_{i,r}(\tau) - \sum_{j \neq i} A_{ij} \hat{x}_{j,r}(\tau) = \varepsilon, \ \forall i, \tau \tag{2.52}$$

which means that (2.42) is met. In the implementation, however, we accept the solution when (2.52) are within a distance $|\varepsilon| > 0$. Thus, the algorithm terminates when the update of the Lagrangian multipliers stays within a bound ε. For convex problems the solution of Algorithm 1 converge to the solution of the centralized problem (2.50).

Result: Find $u_i(k)$ at each cycle k of the distributed MPC method
for $k = 0, \ldots, K_{sim}$ **do**
    each agent i measures $x_i(k), w_i(k)$;
    while $|\hat{\lambda}_{i,r}(\tau) - \hat{\lambda}_{i,r-1}(\tau)| > \varepsilon$ **do**
        for $i = 1, \ldots, n$ **do**
            solve (2.49);
        end
        each agent i communicates $\{\hat{x}_i(\tau)\}_{\tau=k}^{k+K_{\text{pred}}}$ to connected agents;
        for $i = 1, \ldots, n$ **do**
            sub-gradient update (2.51);
        end
        each agent i communicates $\{\hat{\lambda}_i(\tau)\}_{\tau=k}^{k+K_{\text{pred}}}$ to connected agents;
    end
    each agent i implements $u_i(k) = \hat{u}_i(\tau)|_{\tau=k}$;
end

Algorithm 1: Distributed Model Predictive Control

2.4 Discussion

In this chapter, we have reviewed a price mechanism that can be used for optimal control of a network with decision makers. First, we introduced basic concepts from optimization of static problems. Second, we applied the dual-decomposition technique to a set of dynamically coupled problems, and related the solution to the LQR solution. In the end, the method was combined with MPC and sub-gradient iteration, so that constrained problems can be solved in a completely distributed manner. The resulting algorithm is a price mechanism that we will use to coordinate electrical supply-side and demand-side devices in the smart grid in Chapters 4-6.

CHAPTER 3

A New Information Sharing Model for the Smart Grid

This chapter is based on our work presented in [41] and [39]. The main contributions are found in the concept of an information model in the power network, and how to explicitly design the information sharing network taking into consideration the low, medium and high voltage network. The model can be used with a completely distributed MPC algorithm to coordinate decisions in the complex power network. Such a control strategy using a price mechanism is reviewed in Chapter 2. Further, the model handles constraints from different types of electric devices and the network can have any information topology.

3.1 Problem Setting

The Smart (Power) Grid is the most promising solution for the problems presented by increased electrification, and the large-scale introduction of distributed power generation in the power system. In particular, we address the challenge to match local supply and demand in real-time anticipating on the future behavior and only base decisions on local information.

Recall from Chapter 1 that it is widely agreed that a centralized solution scheme for the so-called *optimal control problem* to coordinate a large number of power producers and consumers in the smart grid is too time consuming, because of the computational complexity [30]. This observation motivates us to consider a model with a distributed information structure for scalability.

To avoid a centralized structure, we propose an information network where each agent has local (imbalance) information about the system when they make their decisions. We introduce an information-sharing network, and dynamically couple the end-users information to coordinate decisions in the network. The information at an end-user is a mixture of personal imbalance, and the connected neighbor's imbalances. In a large network, the distance between suppliers and consumers also plays a role: it is more energy efficient to buy from a close-by end-user, than a far-away end-user. An end-user cannot exchange imbalance information with everybody, but bargains directly with a subset of all end-users in the network. This motivates the choice of information network topology. The idea is that the system, as a total, reaches the same balance as if it could bargain with all end-users directly, but now there is an ordering by information distance to neighbors of who an end-user buys his power from: if the power is available at the direct neighbors, the end-user will buy from this neighbor, and the power coordination is done locally. In the case that an end-user needs to buy from a neighbor that is not a direct neighbor, he must bargain through his neighbor's neighbor connections until an end-user wants to sell.

The information network, is made up by a subset of the end-users in the power network, and is connected to the overall power system. This means that there is also a power exchange between the sub-network and the external network. However, this exchange is not modeled explicitly, but the objective is formulated as if the members of the information network are forming a closed grid, minimizing the imbalance, meaning that the power exchange with the external network is minimized. Then, after the actions are taken, we assume that the excess or shortage of power is taken care of by the external network. The control goal is the supply demand balance at a market level within the information network. We stress that the end-users are virtually connected to the information network, while they are physically connected to the power network. Therefore, the information network does not need to have the same topology as the power network, but it can have any desired topology.

Recall from Chapter 1 that the Optimal Power Flow (OPF) problem is solved to find the optimal power generation given the line power constraints. This is a steady state optimal control problem. Even though we are taking a different perspective in our problem, it is clear that the two problems influence each other. The objective of the OPF is to minimize generation cost while the balance problem is included as a hard constraint, while our objective in contrast, is to minimize the power imbalance in the network by coordinating decisions at the end-user level. Coordinating decisions in the information network will influence the control of central power plants, and thus the OPF problem.

Our proposed information sharing model facilitates distributed decisions of dynamically coupled prosumers in a Smart Grid with input constraints. In Chapters 4-6, we take into account forecast about future behavior when the decision is made. If the end-user receives local real-time information concerning the system's status, possibly in the form of prices, he can make decisions for when to turn on or off his demand, production or storage of electric power such that both the end-users and the overall system benefits.

3.2 Information Network Model

In this section we develop our model for the coordination of power production and consumption in a multi-producer multi-consumer Smart Grid. The goal for the agents is to minimize the power imbalance in the network, which corresponds to adjusting their demand and production to balance the network.

3.2.1 System Description

We start by describing the dynamics of power imbalance $\tilde{x}_i(k) \in \mathbb{R}$ of an agent $i = 1, \ldots, n$ in a network of n agents at time k. An agent (prosumer) is for example a household with a μ-CHP system and other electric devices such as washing machines, freezers and batteries of electric cars, where the electrical production and demand can be adjusted or shifted in time. Each agent has a power demand $d_i(k) \in \mathbb{R}_+$ and a power production $p_i(k) \in \mathbb{R}_-$, where $\mathbb{R}_+$ and $\mathbb{R}_-$ includes zero, c.f. the Notations Chapter. Further, we distinguish between flexible power demand $d_{f,i}(k) \in \mathbb{R}_+$ and production $p_{f,i}(k) \in \mathbb{R}_-$ and external power demand $d_{e,i}(k) \in \mathbb{R}_+$ and production $p_{e,i}(k) \in \mathbb{R}_-$. These variables are related by

$$\begin{aligned} d_i(k) &= d_{f,i}(k) + d_{e,i}(k), \\ p_i(k) &= p_{f,i}(k) + p_{e,i}(k). \end{aligned} \tag{3.1}$$

In this model, the external demand and production are all the power demand and production at the agent that cannot be adjusted. The external signal can be measured at each time step k, while the flexible demand and production can be adjusted by the agent.

The agent decides when to turn on or off the flexible power devices, and how much to ramp them up or down. This means that the agent chooses the change in flexible power demand $u_{d,i}(k) \in \mathbb{R}$ and production $u_{p,i}(k) \in \mathbb{R}$, given by

$$\begin{aligned} u_{d,i}(k) &= d_{f,i}(k+1) - d_{f,i}(k), \\ u_{p,i}(k) &= p_{f,i}(k+1) - p_{f,i}(k). \end{aligned} \tag{3.2}$$

At the same time, the agent measures the change in external demand $w_{d,i}(k) = d_{e,i}(k) - d_{e,i}(k-1)$ and production $w_{p,i}(k) = p_{e,i}(k) - p_{e,i}(k-1)$. The physical *power imbalance* $\tilde{x}_i(k) \in \mathbb{R}$ of agent i is given by the imbalance at the previous time-step plus the change in flexible power $u_i(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} u_{d,i}(k \\ u_{p,i}(k) \end{bmatrix}$ and change in external power $w_i(k) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} w_{d,i}(k) \\ w_{p,i}(k) \end{bmatrix}$

$$\tilde{x}_i(k+1) = \tilde{x}_i(k) + u_i(k) + w_i(k), \tag{3.3}$$

where $i = 1, \dots, n$ and $\tilde{x}_i(k), u_i(k), w_i(k) \in \mathbb{R}$. In (3.3), each agent i keeps track of its own imbalance.

Remark 3.2.1. *For simplicity we keep all imbalance, production and demand at an agent in variables $\tilde{x}_i(k), u_i(k), w_i(k) \in \mathbb{R}$, but the model is extendable to other cases. As an example, the production and demand can be kept separate and thus we would have $\tilde{x}_i(k), u_i(k), w_i(k) \in \mathbb{R}^2$.*

In order for an agent to contribute to the local balancing of power by selling or buying power from neighbors, the agent requires some information about the overall power situation in the network. This could be achieved by communicating with one agent who keeps track of the total imbalance in the network. However, to avoid a centralized structure we introduce the state $x_i(k)$ which represents information about the imbalance of agent i, and depends also on information about imbalance of neighboring agents. We introduce a virtual information network, so that each agent has local information about the system when making the decision. The topology of the information network specifies which subset of agents agent i exchanges information with. Agent i's set of information neighbors N_i is given by

$$N_i \subseteq \{1, \dots, n\} \backslash \{i\}, \tag{3.4}$$

where the agent i itself is excluded.

We include the chosen information topology in our dynamic model by adjusting information weights A_{ij}, $i, j = 1, \dots, n$ in the coupling between the agents' notion of imbalance in the system. The model for the *imbalance information* $x_i(k) \in \mathbb{R}$ at agent i is given by

$$x_i(k+1) = A_{ii}x_i(k) + \sum_{j \in N_i} A_{ij}x_j(k) + u_i(k) + w_i(k), \tag{3.5}$$

where A_{ii} weighs the power imbalance information of agent i itself, and A_{ij} weighs the information received from its neighbors $j \in N_i$. We choose the initial values of $x_i(0)$ to be the real physical imbalance of agent i at the initial time, i.e. $x_i(0) = d_i(0) - p_i(0)$ and $w_i(0) = 0$. As time evolves, information spreads through the network through the neighboring agents N_i. In this way close-by information neighbors can react faster to a change in external power $w_i(k)$ than information neighbors further away. In Figure 3.1 agent 2 is a close by information neighbor of agent 1 while agent 5 is a far away information neighbor of agent 1.

Remark 3.2.2. *When $x_i(k)$ is a scalar, we know from Chapter 2 that this will result in one price (Lagrangian multiplier) associated with each agent. If one wishes to associate one price with the power demand and one price with the power production, the power demand and production information $x_{d,i}(k), x_{p,i}(k)$ needs to be kept separated and $x_i(k) = \begin{bmatrix} x_{d,i}(k) \\ x_{p,i}(k) \end{bmatrix}$. Another choice is to let $x_i(k)$ have the length of the number of flexible devices at the agent, in which case each device gets a price associated.*

Notice that in (3.5) the *physical* imbalance enters the system at each agent i through change in flexible power $u_i(k)$ and change in external power $w_i(k)$, where the physical imbalance is included in the *information* about imbalance $x_i(k+1)$.

To write the system in a compact form, we define the vectors

$$
\begin{aligned}
x(k) &= [x_1(k), \ldots, x_n(k)]^\intercal, \\
u(k) &= [u_1(k), \ldots, u_n(k)]^\intercal, \\
w(k) &= [w_1(k), \ldots, w_n(k)]^\intercal.
\end{aligned}
$$

Then the compact form of model (3.5) is given by

$$
x(k+1) = Ax(k) + Bu(k) + w(k), \tag{3.6}
$$

where input matrix $B \in \mathbb{R}_+^{n \times n}$ is the identity matrix in accordance with (3.5), and *information matrix* $A \in \mathbb{R}_+^{n \times n}$ specifies the topology and weighs the information flow in the network. The weights A_{ii}, A_{ij} in (3.5) are the elements of A.

We can also represent the information matrix A in terms of a graph. We define a directed graph $D = (H_n, E_n)$, with n agents. The agent set is given by $H_n = \{1, \ldots, n\}$, and $E_n \subseteq H_n \times H_n$ denotes the edge set. There is an edge in the graph

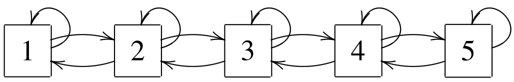


Figure 3.1 – A graph with five agents and a tri-diagonal A matrix. The arrow from agent i to agent j indicates the direction of information flow. Self-loops come from diagonal elements of A.

whenever information is communicated directly from agent i to agent j, i.e. $A_{ij} \neq 0$ ($j \in N_i$) if and only if $(i,j) \in E_n$. Figure 3.1 shows an example for $n = 5$.

We stress the difference between the physical power network and the virtual information network, represented by the information matrix A. In Figure 3.2, the solid lines represent one possible information network where each agent has one information neighbor $N_i = \{i-1\}$, and the self loops represent that agents take their own imbalance information into account as well. In addition to the information network, all agents $i = 1, \ldots, n$ are physically connected to the power grid, which is illustrated by the dashed lines in Figure 3.2. This means that the power demand and production at each agent will affect the overall power imbalance in the system $\sum_{i=1}^{n} \tilde{x}_i(k)$.

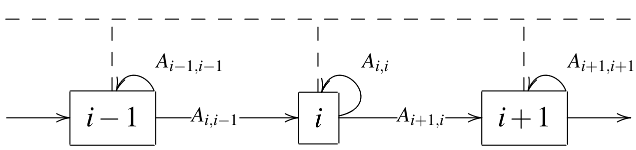


Figure 3.2 – Three agents which are both connected in the power grid, represented by dashed lines, and in one possible information network, represented by solid lines.

We relate the imbalance information to the physical power imbalance by proper choices of the weights in the information matrix A. First of all, when all input is zero

$$w(k) = u(k) = 0, \; \forall\, k \geq 0 \tag{3.7}$$

we require the elements A_{ij} to be such that the total imbalance information in the network is conserved. Thus

$$\sum_{i=1}^{n} x_i(k+1) = \sum_{i=1}^{n} x_i(k), \;\; \forall\, k \geq 0, \tag{3.8}$$

when (3.7) holds. Secondly, we only consider non-negative elements in the A matrices, since we view the imbalance information as a quantity that we want to divide between the agents. Third and finally, for stability of the uncontrolled system, we require the spectral radius of A to be less than or equal to one.

Condition (3.8) implies the following straightforwardly obtained requirement:

Proposition 3.2.1. *The column sums of information matrix A are equal to one, i.e.,*

$$\sum_{i}^{n} A_{ij} = 1, \ j = 1, \dots, n. \tag{3.9}$$

Proof. This is readily checked by substituting (3.5) into the left hand side of (3.8) for $w_i(k) = u_i(k) = 0$ for all i and k, since $A_{ij} = 0$ when $j \notin N_i$. ∎

Corollary 3.2.1. *If A is an irreducible, see e.g. [56] for definition, non-negative matrix and (3.9) is valid, we are ensured that the spectral radius is one.*

Proof. Since A is a stochastic matrix, the result follows directly from the Perron-Frobenius Theorem, see e.g. [56]. ∎

Corollary 3.2.1 implies that we must choose our information graph to be strongly connected, see e.g. [65], so that the system (3.6) is stable when $w(k) = u(k) = 0$ for all k. Loosely said: there is a path in the communication graph from any agent to any agent. R1-R4 sum up the requirements on the information matrix.

- R1 $A_{ij} \neq 0$ if and only if information is exchanged from agent j to agent i.
- R2 All weights are non-negative: $A_{ij} \geq 0, \ i, j = 1, \dots, n$.
- R3 All columns sum up equal to one: $\sum_{i=1}^{n} A_{ij} = 1, \ j = 1, \dots, n$.
- R4 The graph corresponding to information matrix A is strongly connected [65].

Notice that R1-R4 still leave quite some freedom to choose the weights in A. The topology of Figure 3.2 can be captured, but also other possible topologies with information exchange between more neighbors can be captured.

Remark 3.2.3. *The current power network can be captured in the information matrix A, as information neighbors are physically virtual neighbors. By choosing physically nearby neighbors (in the power network) as information neighbors, local production for local demand can be stimulated.*

We assume that the overall power shortage (excess) in the system is imported from (exported to) an external network. We do not include this explicitly in our model, but in Chapters 4 - 6, we will formulate our objective with the aim to minimize this exchange.

In the end, we relate the physical imbalance at the agents to the imbalance information at the agents. With the above requirements R1-R4 on the information weights A_{ij} and initial conditions $x_i(0) = \tilde{x}_i(0) = d_i(0) + p_i(0)$, summing over all agents for (3.3) and (3.5) results in the relation

$$\sum_{i=1}^{n} x_i(k) = \sum_{i=1}^{n} \tilde{x}_i(k), \; \forall\, k \geq 0, \tag{3.10}$$

which means that the total imbalance information in the network is equal to the total power imbalance in the network even though $x_i(k) \neq \tilde{x}_i(k)$ at an agent level.

3.2.2 Network Objective

For each agent i, given an imbalance $x_i(k)$ and change in flexible power $u_i(k)$, we associate an objective function $V_i(x_i(k), u_i(k))$. We require that the function has its minimum when the imbalance in the network is minimized. In particular we choose the objective function at time k to be

$$V_i(x_i(k), u_i(k)) = R_{ii} x_i^2(k) + Q_{ii} u_i^2(k), \tag{3.11}$$

where the weights $R_{ii}, Q_{ii} > 0$ indicate the relative importance of each agent. A hospital can be given more importance than a household, and one electric device might be more costly to operate than another electric device.

The network objective $V(x(k), u(k))$ at time k is assumed to be the sum of the individual objective functions

$$V(x(k), u(k)) = \sum_{i=1}^{n} V_i(x_i(k), u_i(k)). \tag{3.12}$$

Three reasons for introducing (3.11) and (3.12) in the form that we do are; 1: the function have the minimum in zero in order to balance power supply and demand, 2: from an agent's perspective it is better to have a minimum per agent rather than minimizing the square the sum of the imbalance of the agents, 3: from an optimal control and computational perspective quadratic cost functions are motivated by convexity and differentiability arguments.

3.3 Constructing the Information Matrix

We have argued that the information topology and the power network topology do not need to be the same. In this section, however, we will build an information matrix A motivated by the physical structure of the current power grid to demonstrate that this is possible as well. Figure 3.3 displays a schematic overview of the Dutch power infrastructure, where a circle represents an agent. Then we continue with a more sparse information topology, and even a possible change in the power grid structure itself, using our network model.

There is a high voltage (HV), a medium voltage (MV), and finally a low voltage (LV) transmission network to which the agents are connected. The reason for this layered structure is to minimize losses in the power lines. The losses in the lines increase with the current. By increasing the voltage the current for transporting the same amount of energy can be decreased.

However, households in The Netherlands need to be connected to a grid of 220 Volt. To transform down the voltages between the different layers of transmission networks there are transformer stations, indicated as squares in Figure 3.3. These transformer nodes have load constraints, if the load over the transformer is too high a power black out occurs. Since network components have to be designed for the peak-load, it would be good for the network if the load is kept close to a target value. This way the network is used in a more efficient way. The circles in Figure 3.3 represents the n number of agents described in Section 3.2.1. These LV connections are prosumer households.

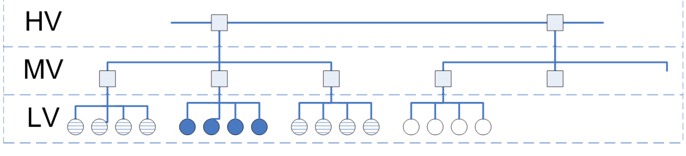


Figure 3.3 – Schematic representation of the current power network in The Netherlands. The squares represent transformer nodes, and the circles represent end agents.

In order to construct one row in the information matrix $A \in \mathbb{R}^{n\times n}$, we start by looking at one agent i, represented by a dark colored circle i in Figure 3.3, and discuss how to weigh the neighbors $j \in N_i$. Suppose that a change in power demand occurs at agent i present in a street in city X, for instance by turning on a washing machine. Then, it is natural that if agent i has available production capacity, he should ramp up the production so that the additional power imbalance added to the network by agent i is as low as possible. It is also clear, that physical neighbors of agent i in the same street and connected to the same LV network, should ramp up their production before a distant physical neighbor in a far away city Y would do so. This is because we wish to keep the load over a transformer station low, or at a target value. With these considerations, we will first assume a full information matrix, i.e. $N_i = \{1,\dots,n\}\backslash\{i\}$ for all $i = 1,\cdots,n$. In this example, we assume that there is one HV network, with $\nu \in \mathbb{N}_+$ HV-MV transformer stations. Each of the ν MV networks has $\eta_i \in \mathbb{N}_+$ MV-LV transformer stations, where η_i varies from MV network to MV network. Each LV network has $m_i \in \mathbb{N}_+$ LV connections, i.e. m_i number of agents. The number of agents in the system is therefore $n = \sum_i m_i$.

Suppose agent i is represented by one of the dark colored circles in Figure 3.3, and he weighs his own power imbalance with a factor $\alpha \in \mathbb{R}_+$. He weighs all neighbors connected to the same LV transformer, represented by the remaining dark circles in Figure 3.3, by $\beta \in \mathbb{R}_+$. Then choosing $\alpha > \beta$, reflects that agent i reacts more strongly on his own imbalance than his LV neighbors. Or equivalently, agent i finds it more important to react to the change of his own state, than to react to changes in neighboring agents' states. Next, agent i weighs all the agents at a different LV network, but the same MV network, represented by striped circles in Figure 3.3, by a weight $\varepsilon \in \mathbb{R}_+$. In the end distant agents, maybe located in the other side of the country, at the same HV network but different MV network is given a lower weight $\varepsilon' \in \mathbb{R}_+$. These agents are denoted by white circles in Figure 3.3. Agent i organizes the relative importance of different type of neighbors by $\alpha > \beta > \varepsilon > \varepsilon'$.

If we assume that all agents in the power network make the same choices, we can build an information matrix $A \in \mathbb{R}^{n\times n}$ where all column sums equal one, by blocks of sub-stochastic matrices $A^{(i)}_{LV}, A^{(i)}_{MV}$, and one matrices $G^{(i)}_{LV}, G^{(i)}_{MV}$ of proper dimensions by

$$
A_{LV}^{(i)} = \begin{bmatrix} \alpha & \beta & \cdots & \beta \\ \beta & \alpha & \cdots & \beta \\ \vdots & & \ddots & \vdots \\ \beta & \beta & \cdots & \alpha \end{bmatrix} \in \mathbb{R}^{m_i \times m_i} \tag{3.13}
$$

$$
A_{MV}^{(i)} = \begin{bmatrix} A_{LV}^{(1)} & \varepsilon G_{LV}^{(2)} & \cdots & \varepsilon G_{LV}^{(\eta_i)} \\ \varepsilon G_{LV}^{(1)} & A_{LV}^{(2)} & \cdots & \varepsilon G_{LV}^{(\eta_i)} \\ \vdots & & \ddots & \vdots \\ \varepsilon G_{LV}^{(1)} & \varepsilon G_{LV}^{(2)} & \cdots & A_{LV}^{(\eta_i)} \end{bmatrix} \in \mathbb{R}^{\eta_i \times \eta_i} \tag{3.14}
$$

$$
A = A_{HV} = \begin{bmatrix} A_{MV}^{(1)} & \varepsilon' G_{MV}^{(2)} & \cdots & \varepsilon' G_{MV}^{(\nu)} \\ \varepsilon' G_{MV}^{(1)} & A_{MV}^{(2)} & \cdots & \varepsilon' G_{MV}^{(\nu)} \\ \vdots & & \ddots & \vdots \\ \varepsilon' G_{MV}^{(1)} & \varepsilon' G_{MV}^{(2)} & \cdots & A_{MV}^{(\nu)} \end{bmatrix} \in \mathbb{R}^{\nu \times \nu} \tag{3.15}
$$

where $\mathbb{R}^{\eta_i \times \eta_i}$ and $\mathbb{R}^{\nu \times \nu}$ are the dimension of the number of blocks in the corresponding $A_{MV}^{(i)}$ matrices and A matrix. The dimension of A in terms of agents is $\mathbb{R}^{n \times n}$, and the parameters $\alpha, \beta, \varepsilon, \varepsilon'$ are chosen such that $\sum_j A_{ij} = 1$, meaning that no information is lost.

In this setup, the A matrix (3.15) is full, meaning that agent i receives information from all neighbors in the network. However, with our way of modeling (3.5), we are not restricted to choose the physical neighbors to be communication neighbors. We can freely choose our communication neighbors, which has the consequence that neighbors in an LV transportation grid may have different information about the situation in the grid, when they make decisions about the power they will produce (buy). If no agent implements any action, all agents in the network will converge to see the same imbalance if A is chosen according R1-R4 in Section 3.2.1.

Other Possible Grid Configurations

By taking into account all elements in (3.15) the information exchange is not easily manageable for a large system. A strength of the framework is that we can set elements in the A matrix to zero, representing information that is not important to an agent. We then have a smaller number of direct neighbors, and thus less information exchange. The remaining elements of A should be updated accordingly, as the model is valid for any structure of the A matrix such that Proposition 3.2.1 holds, $A_{ij} \in \mathbb{R}_+$, and A is a matrix such that the system is stable.

Above, the current network structure was captured in the model, but with a large enough share of distributed generation the hierarchical structure in Figure 3.3 may not be necessary. One possible configuration is given in Figure 3.4. In the extreme case, a large number of agents can form a stand-alone power grid.

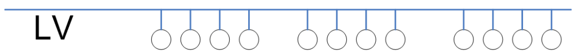


Figure 3.4 – Possible grid topology, where a large cluster of agents are locally balanced.

The corresponding connectivity matrix for Figure 3.4, can be built up by matrix (3.13). The A_{LV} matrix has the dimension of the number of neighbors connected to one LV transformer.

In Figure 3.4 the physical and the information structure is still the same. However, we can also adjust the number of information neighbors, so that the information infrastructure does not correspond to the physical infrastructure in the network. The physical structure may be as in Figure 3.4 while the information structure is a chain as in Figure 3.5. In this example, all agents except the first and the last, then take exactly two neighbors into account in the update of their state equation, while they are all connected in the power network. Notice that the information exchange does not need to be bidirectional. When the graph is strongly connected, and if no agent implements any action, all agents in the network will now still converge to see the same imbalance. An agent i in city X then only needs to obtain information about the change in demand from a few neighbors in his own street. If there is a change in demand at an agent j in a distant city Y, this information will only reach agent i if the imbalance is not already taken care of.

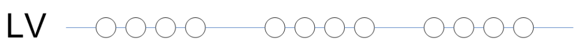


Figure 3.5 – Possible information structure.

Claim 3.3.1. *By choosing physically close neighbors as information neighbors, local production for local demand is stimulated. This is because the agents that re-*

ceive the information about a change in demand the fastest, will be able to react to this change first. Consequently transportation losses in the power grid are avoided.

3.4 Discussion

In this chapter, we have designed an information sharing model that can be used together with a distributed MPC method to achieve supply-demand balance in the power network. We couple the agents dynamically through their notion of power imbalance information. In Section 3.2 we have given some necessary conditions for the design of the information matrix A, and we have suggested possible network topologies. The model has the freedom to take into account different capacities for each agent, and the agents may be weighed with different importance. It is set up such that it can be used together with different models for demand side and supply side devices. Further, it is possible to include storage, e.g. electric car batteries, to be coordinated using the network model described in Section 3.2.

In Chapters 4 - 6 we combine the developed network model with models for specific electric devices, and with the distributed MPC algorithm reviewed in Chapter 2.

CHAPTER 4

Control of a Network with Multiple Electricity Producers

In this chapter, we consider a network of agents with an electricity production that can be controlled and an electricity demand that cannot be controlled. The production must be coordinated amongst the users in order to balance the demand. First we combine the theory reviewed in Section 2.2 with the information sharing model for multiple electricity producers in Chapter 3 as presented in our paper [36]. Second, to include non-convex on-off constraints and range constraints on the power production, the distributed MPC method reviewed in Section 2.3 is combined with the network model in Chapter 3, as presented in our papers [37] and [39].

4.1 Problem Setting

In future power networks, a large scale introduction of micro Combined Heat and Power (μ-CHP) systems is expected. A μ-CHP unit, in a household, produces heat that can be used for private consumption, and power that can be injected in the power network. Since both heat and power can be used, the overall energy efficiency of the μ-CHP is 1.24 times higher compared to a traditional power plant in combination with a gas-fired boiler [23]. The market potential is considered high [63], and the μ-CHP systems fueled on gas are of particular interest in countries like The Netherlands where the gas grid is dense.

The power output of the μ-CHP can be controlled, even though it is subject to several operational constraints. An overview of control strategies for the μ-CHP is given in [20]. In [23] a Model Predictive Control (MPC) approach for the modeling and control of a μ-CHP unit with demand response is presented. However, the power production from the μ-CHP influences the power balance in the network. It is therefore of importance to consider how a large number of μ-CHPs can influence the real-time balance of power in the network.

Recall from Chapter 1 that within the setting of Smart Grids, the households (agents) have the potential to contribute to the balance of the system [31]. We therefore foresee a shift towards a situation where a large number of smaller agents have more market power if they can coordinate their decisions. The motive can be both economical and environmental, as we assume a better price for the power when the resources are used more efficiently, and it is better for the environment when the energy losses are reduced.

In this chapter, we study a possible optimal control scheme based on a price mechanism, for a network of μ-CHPs. We set two requirements for the scheme. First, in a large scale power network there are power losses in the transportation lines. Therefore, the objective of the control scheme is to map local power production to local consumption. Secondly, the control scheme has to scale well. We therefore look for a distributed approach, meaning that each agent make its own decision whether its μ-CHP should be on or off based on local information.

To avoid a centralized structure, we use the information network proposed in Chapter 3 where all agents have local (imbalance) information about the system when they make their decisions. In a large network, the distance between suppliers and consumers is playing a role. An agent is not exchanging imbalance information with everybody, but bargains directly with a subset of all agents in the virtual information network according to the information structure.

The agents in the virtual information network is an isolated subset of the households connected in the power grid. The goal is to minimize the difference between power demand and production in the information network by locally determining the off (on) state of the μ-CHPs. We assume that external parties are responsible for the overall power balance in the network, but the information network is contributing to the overall balancing. In this chapter, the comfort-levels of the agents are not altered. Thus, we do not consider here that the demand will be influenced, this is however done in Chapter 6.

Our information sharing model from Chapter 3 is combined with the distributed control method reviewed in Chapter 2. In this way, the agents' decisions are made in a completely distributed way. This strategy, based on dual-decomposition, is applied to control a formation of vehicles [51]. By exchanging only prices, the vehicles hold the desired position. This is also an attractive idea for control of decentralized power generation. We adopt these methods and apply them to the power supply-demand balance setting.

4.2 Prosumers with Unconstrained Electricity Production

The decomposition method described in the Section 2.2 fits well with the vision of distributed control of components in the electricity grid. We have a network of agents that can turn on and off their μ-CHP. It is assumed that the agents get cheaper electricity if they can coordinate their decisions in such a way that the net current flow is at a target value. If the net load is high, the electricity price is high. We expect a better average price for the agent if peaks in the net load are avoided.

All agents make their decisions to turn on and off devices, based on different information concerning underlying uncertainties about their electricity needs. We want to see how distributed feedback controllers can be applied to a network of agents with a μCHP present. In this setting the agents exchange price information to their neighbors in order to coordinate their decisions.

In this section, we do not include any input constraints. Therefore, the theory in Section 2.2 and the model in Chapter 3 are combined straightforwardly. We are interested in how this strategy performs.

4.2.1 System Description

Since we only consider the flexible supply-side device and external demand-side devices here, we adjust the notation from Chapter 3 in the following way. In the rest of this chapter we will use $d_{f,i}(k) = 0$, $d_{e,i}(k) = d_i(k)$, $p_{f,i}(k) = p_i(k)$, and $p_{e,i}(k) = 0$.

Thus, the system consists of n agents (prosumers), which represent for example households with a μ-CHP or larger prosumers such as a hospital with a CHP. At the discrete time-step k agent i has a power demand $d_i(k) \in \mathbb{R}_+$ and a power production $p_i(k) \in \mathbb{R}_-$. The agents are dynamically coupled to a subset N_i of the virtual neighbors in the network, through (3.5) which we repeat here

$$x_i(k+1) = A_{ii}x_i(k) + \sum_{j \in N_i} A_{ij}x_j(k) + u_i(k) + w_i(k), \tag{4.1}$$

where the state $x_i(k)$ of household i is this agent's information about the net power imbalance, and $w_i(k) = d_i(k) - d_i(k-1)$ is the change in power demand. Recall from Chapter 3, that the information about imbalance has to be distinguished from the physical imbalance at the agent (3.3). The demand is an external signal that is only measured at each time step k, which means that we are not altering the comfort-level of the agents. It is the production that can be adjusted by the agent.

Therefore, the decision to be made at each agent is how much to ramp up (down) the power production $u_i(k)$, where

$$u_i(k) = p_i(k) - p_i(k-1). \tag{4.2}$$

As before, $x_i(k)$ in (4.1) is a combination of the household's own imbalance and its information neighbors' imbalance, where the weights are specified by the information matrix A. The information matrix has to meet requirements R1-R4 given in Section 3.2.1.

We assume that the overall power shortage (excess) in the system is imported from (exported to) an external network. Therefore, we do not include this explicitly in our model, but we formulate our objective with the aim to minimize this exchange.

Objective Function

The objective is to map local power supply to local power demand. Therefore, our goal is to find the control input $u_i(k)$ for an agent i such that the imbalance $x_i(k)$ becomes zero for all agents $i = 1,\ldots,n$, given the influence from neighbors and physical constraints on the μ-CHP.

For a given imbalance $x_i(k)$ and change in production $u_i(k)$, we associate a cost $V_i(x_i(k),u_i(k))$ to each agent $i = 1,\ldots,n$. In particular we choose $V_i(x_i(k),u_i(k))$ at time k to be

$$V_i(x_i(k),u_i(k)) = R_{ii}x_i^2(k) + Q_{ii}u_i^2(k), \tag{4.3}$$

where the weights $R_{ii}, Q_{ii} > 0$ indicate the relative importance of each agent. If one wishes to make the imbalance of a large industry more important than e.g. a household, one can choose the corresponding weight higher.

The network cost $V(x(k),u(k))$ at time k is assumed to be the sum of the individual costs

$$V(x(k),u(k)) = \sum_{i=1}^{n} V_i(x_i(k),u_i(k)). \tag{4.4}$$

In our optimal control problem, the goal is to find the inputs $u_i(k)$, $i = 1,\ldots,n$, that minimize the overall power exchange with large external power suppliers in the network over an infinite horizon such that the constraints hold. This objective becomes

$$\underset{u}{\text{minimize}} \limsup_{K\to\infty} \frac{1}{K} \sum_{k=0}^{K} V(x(k),u(k)), \tag{4.5}$$

which is such that the network as a total wants to regulate the imbalance to zero at minimal production cost. Three reasons given in Chapter 3 for introducing (4.3) and (4.4) in the form that we do are; 1: the function must have the minimum in zero in order to balance power supply and demand, 2: from an agent's perspective it is better to have a minimum per agent rather than minimizing the square the sum of the imbalance of the agents, 3: from an optimal control and computational perspective quadratic cost functions are motivated by convexity and differentiability arguments.

Remark 4.2.1. *We have assumed that when the total imbalance* $\sum_{i=1}^{n} x_i(k)$ *is positive, the network has to import power from the external network, and when the total agent imbalance is negative the network is a provider of power to the external network.*

4.2.2 Results

Here we investigate how the optimal control method from Section 2.2 can control the decentralized power production to balance electricity usage in a street of houses. In this example losses are not important, but the effect of information constraints is interesting. Recall from Chapter 3 that the idea is to balance the production and consumption within a network of agents, by only sharing information with a subset of the agents in the network. In this setting, an agent in the network is behaving like a fish in its school. With information from a few neighbors, the network of agents can maneuver together in an efficient way.

In this numerical example, the change in power demands are external inputs representing five unique household profiles [49]. The demand patterns are simulated assuming weather conditions of a November month in The Netherlands. It is clear that the average demand per household varies considerably from household to household. A couple in an apartment has a much lower energy consumption than a large family in a stand alone house. Therefore, to capture these differences, the most important characteristics of five different household sizes are captured in the five test patterns. More information about these demand patterns and different electrical user profiles can be found in Appendix A and Figure A.1(a).

In the simulation we used 1440 data points with a resolution of one minute, giving the total of one day. Note that in the control method presented in Section 2.2 the disturbance $w(k)$ was white noise with a given variance σ^2. Thus, here we assume that the input for the change in power demand, $w_i(k)$ in (4.1), from the realistic data can be taken to be white noise.

The information matrix A from (4.1) is given by

$$A = \begin{bmatrix} 0.8 & 0.1 & 0 & 0 & 0.1 \\ 0.1 & 0.8 & 0.1 & 0 & 0 \\ 0 & 0.1 & 0.8 & 0.1 & 0 \\ 0 & 0 & 0.1 & 0.8 & 0.1 \\ 0.1 & 0 & 0 & 0.1 & 0.8 \end{bmatrix}, \tag{4.6}$$

which means that each agent weighs its own imbalance with 0.8 and two neighbors' imbalances by 0.1. This is a circular information network. The corresponding feedback matrix in (2.19) found by (2.21) is given by

$$L = \begin{bmatrix} 0.467 & 0.073 & 0.003 & 0.003 & 0.073 \\ 0.073 & 0.467 & 0.073 & 0.003 & 0.003 \\ 0.003 & 0.073 & 0.467 & 0.073 & 0.003 \\ 0.003 & 0.003 & 0.073 & 0.467 & 0.073 \\ 0.073 & 0.003 & 0.003 & 0.073 & 0.467 \end{bmatrix}. \tag{4.7}$$

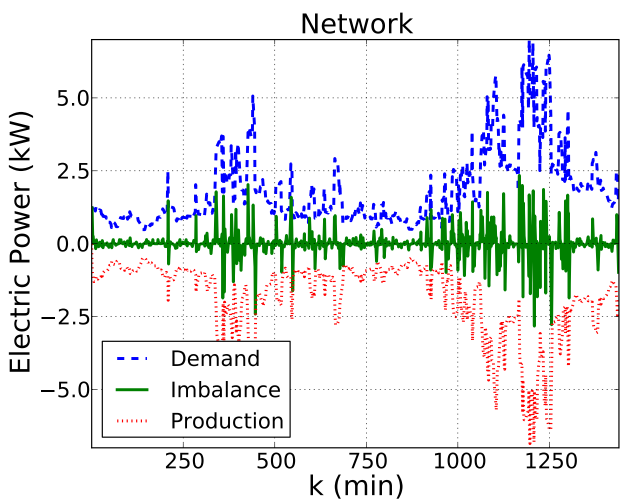


Figure 4.1 – Simulation of five households with realistic delta demand patterns with a resolution of 1 minute. The figure shows the net demand $\sum_i d_i(k)$, the net production $\sum_i d_i(k)$, and net imbalance information $\sum_i x_i(k)$.

Figure 4.1 shows the result for the network of five prosumers. The solid line is the net imbalance $\sum_i x_i(k)$, the dashed line is the net power demand $\sum_i d_i(k)$, and the dotted line is the net power production $\sum_i p_i(k)$. The method regulates the net

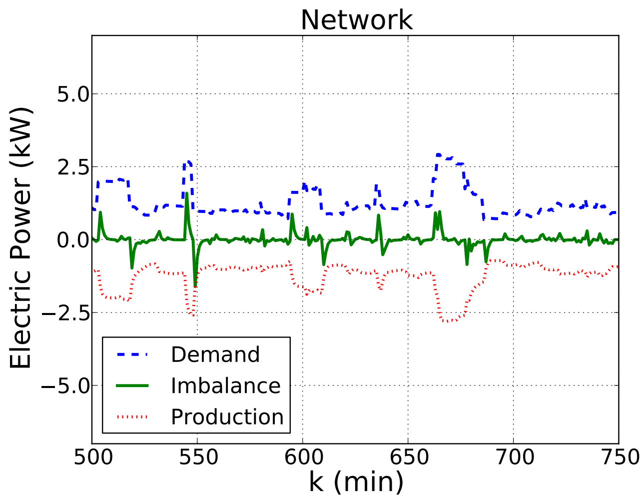


Figure 4.2 – A zoom on the interval $k = 500, \ldots, 750$ in Figure 4.1.

imbalance to zero by adjusting the change in electricity productions $u_i(k)$ in (4.1). Peaks in the imbalance is caused by sudden changes in the electricity demand. By the proposed control, the production is reacting fast to the changes to recover balance in the network. Figure 4.2 is included to show this feature more clearly. The figure provides a zoomed in view of Figure 4.1 on the interval $k = 500, \ldots, 750$ minutes. At $k = 670$ minutes the demand in the network is increasing, resulting in a peak in the imbalance. The production is ramped up so that the imbalance returns to zero shortly after. This is also apparent from Figure 4.1, as we see that the power production mirrors the power demand.

Figure 4.3 shows the result for a single prosumer in the network of five households. For completeness, the results for all five household types are shown in Appendix A Figure A.2. Compared to Figure 4.1, stochastic peaks in the power demand are more present in the imbalance information, see e.g. $k = 270$ minutes. More network configurations were investigated in [14], with comparable results.

The control input, $u(k)$, is not constrained in this numerical example, but in reality there are restrictions on the power output and the on (off) states of a μ-CHP. Here this simulation serves only as a proof of concept.

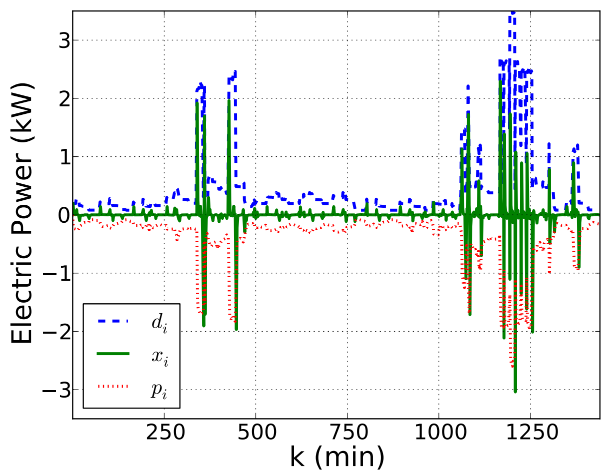


Figure 4.3 – The situation of a single household in the network in Figure 4.1. The demand d_i is often larger than 1 kW. Compared to Figure 4.1 stochastic peaks in the power demand are more present in the imbalance information.

4.2.3 Discussion

The performance of a circular configuration was illustrated for five households, each with a μ-CHP installed and a realistic demand pattern. In this case, the production followed the demand in the network. The power imbalance is steered towards zero locally, by sharing information with two neighbors. This is promising also with respect to a larger network, because a large number of agents can be virtually connected to a low number of closest neighbors. Information from neighbors physically far away from the agent can be set to zero, so that the number of information exchanges in the network is relatively low.

We see, however, peaks in the imbalance in Figure 4.1. This is due to the fact that changes in the demand are measured at each time-step, and thus there is a delay when the change in demand is measured till it is included in the imbalance (4.1). A large number of agents with local balancing has great potential with respect to eliminating these peaks. A strength with a large network is namely that when a large number of unique balanced patterns are summed up, stochastic peaks are less present. This effect can be seen even going from one prosumer in Figure 4.3 till five prosumers in Figure 4.1.

To return to the main question posted in Section 4.2, we have seen that by embedding the electrical power grid in the framework of dual-decomposition given in Section 2.2, distributed control of decentralized power generation can be achieved. This is promising with respect to computational complexity. Note also that the implemented control is optimal for a given A matrix, and thus the feedback matrix L computed by (2.21) is full. If a structure is imposed on the feedback matrix L, a sub-optimality bound can be given as in [52].

However, we have not included any input constraints in this section. Typically, a μ-CHP unit can produce power in the range 0.3kW - 1kW. Some μ-CHP models even have a minimum on (off) time after the machine has been turned on (off). Figure 4.3 shows that the optimal production is often higher than 1kW and lower than 0.3 kW at a single agent. Thus, these values cannot be implemented by a real μ-CHP. If we include this input constraint, it is clear that it is useful with our information sharing model. In this case, when an agent has a higher demand than what the μ-CHP can produce, the extra power can be provided by neighbors who are able to produce more than their demand at that time.

On the other hand, letting the input be without constraints is not a completely unrealistic assumption, since with battery storage and demand planning in time, the total demand could be covered by the μ-CHPs. Five one kW electric μ-CHP systems in the network are able to produce five kW on full power. When compared to the net demand in Figure 4.1, we see that only a few peaks in the interval $k = 1000, \ldots, 1250$ minutes exceed five kW.

Further, we also expect that agents can have some prediction about their own power demand in the near future. This prediction can also be considered when the control input is calculated. A natural next step is, therefore, to extend the model in Section 4.2.1 to also include constraints from the μ-CHP, and combine the new model with the distributed MPC method reviewed in Section 2.3. In the MPC framework constraints and predictions are included in a natural way.

4.3 Prosumers with μ-CHPs Including on-off Constraints and Power Modulation

In Section 4.2, a formulation of the optimization problem with μ-CHPs in a network was introduced. Then the dynamic price mechanism of Section 2.2 was applied. However, the consideration of technical constraints from the μ-CHPs was lacking. Due to the constraints, MPC is a useful technique to solve the optimal control problem, e.g. [46]. It is also a framework where predictions and forecasts of the agents behavior are naturally included.

The main contribution in this section is the embedding of the μ-CHPs in the power network using the fully distributed MPC setting in Section 2.3 together with our information sharing model. The method includes forecasts, real-time optimal control and distributed decision making to achieve a power balance. We examine practical control considerations due to the on (off) restriction of the μ-CHP. Two different strategies for inclusion of the on (off) characteristics of the μ-CHPs are presented. One strategy uses a quadratic program to solve the problem, while the other uses a mixed integer quadratic program to find the solution. The methods are tested with realistic power demand patterns from different types of households, and the scalability to a network of 1000 agents is considered. This section is based on our papers [37] and [39].

We use the same system as in Section 4.2.1, but expand it to also include input constraints.

4.3.1 Physical Constraints from the μ-CHP

The μ-CHPs have physical restrictions that constrain the control input $u_i(k)$ that can be implemented at each agent. The change in power production $u_i(k)$ is related to the production $p_i(k)$ through a dynamic equation. Depending on its implementation, the change in production $u_i(k)$ can be found in $p_i(k)$ at the same time-step k or later. We choose to model this relation as in (4.2). Thus we have

$$u_i(k) = p_i(k) - p_i(k-1). \tag{4.8}$$

In Section 4.2, the production $p_i(k)$ could take on any value. However, a real μ-CHP can only deliver power in a range between a maximum and a minimum value. Here, we therefore include a range constraint that reflects that there is a maximum power $p_{i,\max} > 0$ and a minimum power $p_{i,\max} > p_{i,\min} \geq 0$ that the μ-CHP can deliver. Here $p_{i,\max}, p_{i,\min} \in \mathbb{R}_+$ are constant scalars, but may have different values for each agent i depending on the size of the μ-CHP.

Ultimately we want the power output to be either zero or within a range. Hence, the production set is defined by

$$P_i(k) = \{p_i(k) | -p_{i,\max} \leq p_i(k) \leq -p_{i,\min}\} \cup \{0\}, \tag{4.9}$$

for all $k > 0$, and the production is defined to be negative compared to the demand in Section 3.2.1. This is a *non-convex* set when $p_{i,\min} > 0$. In practice, $p_{i,\min}$ will always be strictly larger than zero because the μ-CHP cannot produce infinity small amounts of power. Such non-convexities are a potential challenge when solving optimal control problems. In Section 4.3.3, we suggest two different methods to deal with the non-convexities problem. In one implementation we keep the constraints convex, which means that we do not exclude the gap between zero power output and the minimum power output from the μ-CHP in the optimization problem. If the solution of the optimization problem lies in this gap outside the physical power range of the device, the physical value that is closest to the solution of the optimization problem is implemented. The other method is to include the binary constraints explicitly in the optimization problem.

In addition to the non-convex constraint, we include constraints that reflect the presence of a minimum run-time $T_{i,\text{on}}$ (minimum off-time $T_{i,\text{off}}$) where we require that the μ-CHP stays on (off) for at least $T_{i,\text{on}}(T_{i,\text{off}})$ time steps after the machine was turned on (off). This means that $\{0\}$ is excluded from the set (4.9) for $T_{i,\text{on}}$ time-steps after the μ-CHP is turned on. Similarly $\{p_i(k)| - p_{i,\max} \leq p_i(k) \leq -p_{i,\min}\}$ is excluded from the set (4.9) for $T_{i,\text{off}}$ time-steps after the μ-CHP is turned off.

4.3.2 Centralized and Distributed MPC Problem

In line with the theory presented in Section 2.3, we will no longer aim to solve (4.5). Instead, the task is to choose when to turn on (off), and how much to ramp up (down) the μ-CHP, so that the corresponding finite horizon cost is minimized at eat time step k.

The centralized problem is, therefore, given by

$$\begin{aligned}
\underset{\hat{u}}{\text{minimize}} \quad & \sum_{\tau=k}^{k+K_{\text{pred}}} V(\hat{x}(\tau), \hat{u}(\tau)), \\
\text{subject to} \quad & \hat{x}(\tau+1) = A\hat{x}(\tau) + \hat{u}(\tau) + \hat{w}(\tau), && \tau = k, \ldots k + K_{\text{pred}} - 1 \\
& \hat{x}(\tau)_{\tau=k} = x(k), \\
& \hat{x}(\tau) \in X, \ \hat{u}(\tau) \in U, && \tau = k, \ldots k + K_{\text{pred}}, \\
& \mu\text{-CHP constraints from Section 4.3.1},
\end{aligned} \tag{4.10}$$

where $V(\hat{x}(\tau), \hat{u}(\tau))$ is defined in (4.4), $\hat{w}(\tau)$ is an external signal, X is a convex set as in Section 2.3.1, but U is no longer convex due to the operational constraints from the μ-CHP. The distributed problem is to solve iteratively the price iterations

$$\hat{\lambda}_{i,r+1}(\tau) = \hat{\lambda}_{i,r}(\tau) + \gamma_{i,r}[\hat{v}_{i,r}(\tau) - \sum_{j\neq i} A_{ij}\hat{x}_{j,r}(\tau)], \tag{4.11}$$

and the following decoupled problems

$$\begin{aligned}
\underset{\hat{u}_i,\hat{v}_i}{\text{minimize}} \quad & \sum_{\tau=k}^{k+K_{\text{pred}}} \Big(V_i(\hat{x}_i(\tau),\hat{u}_i(\tau)) + \hat{\lambda}_i(\tau)\hat{v}_i(\tau) - \sum_{j\neq i} \hat{\lambda}_j(\tau) A_{ji}\hat{x}_i(\tau) \Big), \\
\text{subject to} \quad & \hat{x}_i(\tau+1) = A_{ii}\hat{x}_i(\tau) + \hat{v}_i(\tau) + \hat{u}_i(\tau) + \hat{w}_i(\tau), \;\; \tau = k,\dots k+K_{\text{pred}}-1 \\
& \hat{x}_i(\tau)_{\tau=k} = x_i(k), \\
& \hat{x}_i(\tau) \in X_i, \; \hat{u}_i(\tau) \in U_i, \;\; \tau = k,\dots k+K_{\text{pred}} \\
& \mu\text{-CHP constraints from Section 4.3.1,}
\end{aligned} \tag{4.12}$$

where where $V_i(\hat{x}_i(\tau),\hat{u}_i(\tau))$ is defined in (4.3), $\hat{w}_i(\tau)$ are external signals, X_i is a convex set as in Section 2.3.2, but U_i is no longer convex due to the operational constraints from the μ-CHP. Thus, we determine $u_i(k)$ given the network model and constraints presented in Section 4.3.1. In the following section, we examine difficulties with the distributed implementation due to the gap between $p_{i,\min}$ and zero.

4.3.3 Explicit Inclusion of μ-CHP Constraints

Here we explicitly include constraints from the μ-CHP given in Section 4.3.1, in the decoupled sub-problems given in (4.12). We propose two distinct schemes for including the operational constraints given in Section 4.3.1, i.e. that the μ-CHPs cannot produce very small amounts of power. The first formulation (Problem QP) is convex, which makes it fast to solve with available optimization algorithms and the theory provided in Section 2.3 is valid. However, we will sometimes face solutions from the optimization algorithm that can not be implemented in practice, because the gap between zero and $p_{i,\min}$ is not taken into account. In this case, a sub-optimal solution has to be implemented in the MPC time-step, i.e. either zero or $p_{i,\min}$. The second formulation (Problem MIQP), is closer to reality, as the optimization problem includes the logics of turning the μ-CHP on and off. If the solution exists it will always provide a solution that can be implemented on the μ-CHP. As $p_i(k)$ cannot take its values in a continuous interval, we are not guaranteed to find a solution. Here, this means that we expect to encounter situations where the solution $u_i(k)$ to be implemented oscillates between turn on (off) and stay off (on), and by implementing on or off in MPC time-step the network will produce slightly more or less power than what is optimal.

In both problem formulations, we will find an action that can be implemented in practice at the MPC time step, even though the optimization does not provide such a solution. This means that due to the MPC formulation we can always guarantee that the constraints are met, but due to the gap between zero and $p_{i,\min}$ the solution will be sub-optimal. Since "Problem QP" is the computationally cheapest, we are interested to see whether this problem formulation is useful compared to the "Problem MIQP" formulation. In addition, since the second problem is non-convex we are interested to see if the algorithm still provides solutions.

4.3.3.1 Solving a Quadratic Program (QP)

In this formulation the constraint set $P_i(\tau)$ for the local sub-problems (4.12) is a pre-specified real interval set that is reset at each time-step k. The optimization does not include the off switching during an optimization cycle. Unless further restrictions are known up front, the constrained set is defined for all agents $i = 1, \ldots, n$ over the horizon $\tau = k, \ldots, k + K_{\text{pred}}$ by

$$P_i(\tau) = \{\hat{p}_i(\tau) | -p_{i,\max} \leq \hat{p}_i(\tau) \leq 0\}, \tag{4.13}$$

where the minus sign is defined in Section 3.2.1. If the state of the μ-CHP has changed, we include additional on (off) restrictions in $P_i(\tau)$. The minimum off-time T_{off} after shut down, and minimum on-time T_{on} after start up is ensured by specifying $P_i(\tau)$ at each MPC starting time-step k, before the optimization is performed. We use the $t_i(k)$ counter to keep track of how long the μ-CHP has been off (on). Regions of the set (4.13) are excluded according to

$$P_i(\tau) = \{0\}, \tag{4.14}$$

for $\tau = k, \ldots, k + T_{i,\text{off}} - t_i(k)$ and according to

$$P_i(\tau) = \{\hat{p}_i(\tau) | -p_{i,\max} \leq \hat{p}_i(\tau) \leq -p_{i,\min}\}, \tag{4.15}$$

for $\tau = k, \ldots, k + T_{i,\text{on}} - t_i(k)$, where the counter $t_i(k)$ is set to zero when the μ-CHP changes state, which means that both $p_i(k+1) = 0$ and $p_i(k) \neq 0$. Similarly, $t_i(k)$

is set to zero when $p_i(k+1) \neq 0$ and $p_i(k) = 0$. We stress that production sets can only change after an optimization step is finished, i.e. the on (off) change cannot be taken into account inside the local minimization problem (4.12).

We always converge to the global optimum by iteratively solving the local minimization problem (4.12) and sub-gradient step (4.11), see e.g. [16]. However, when $-p_{i,\min} \geq p_i(k) \geq 0$, in fact we cannot conclude whether we should turn the machine on or off.

Obtaining a Physical Solution

An ad-hoc solution to the problem of the gap between zero and minimum physical production from the μ-CHP, is to choose a threshold $p_{i,\min} > 0$ for the implementation only. If an input $u_i(k)$ is found that would result in a production $p_i(k)$ in the interval smaller than what the μ-CHP can produce $-p_{i,\min} \leq p_i(k) \leq 0$ we have to round the input after the optimization such that $p_i(k) = 0$ or $p_i(k) = -p_{i,\min}$. In this way the notion of turning the μ-CHP on (off) is imposed outside the minimization problem. This way the constraints are met in the distributed MPC algorithm, even if the solution is sub-optimal.

4.3.3.2 Solving a Mixed Integer Quadratic Program (MIQP)

In contrast to *Problem QP* we here introduce a mechanism that resets $t_{\text{on,(off)}}$ *inside* the optimization itself. Thus, we include the logics of turning the μ-CHP on (off). The production sets $P_i(\tau)$ are now non-convex.

We use a Mixed Integer Quadratic Program (MIQP) to solve the optimization problems at each time step k. This type of program that deals with binary variables, is described in [3]. Solving such a combinatorial problem can be time consuming compared to the Problem QP.

Similar to [23], we introduce a binary optimization variable, $\hat{r}_i(\tau)$, which indicates whether the μ-CHP is running or not.

$$\hat{r}_i(\tau) = \begin{cases} 1 & \text{if the } \mu\text{-CHP is on,} \\ 0 & \text{if the } \mu\text{-CHP is off.} \end{cases} \tag{4.16}$$

For correct operation we also need to know if the μ-CHP is turned on or off at a given time step. To keep track of this action we introduce action variables $\hat{a}_i(\tau)$ that is -1 when the μ-CHP is turned off, 0 when there is no change, and 1 when the μ-CHP is turned on.

The equation describing the relation between the run state of the μ-CHP and the action taken at the given time-step is given by

$$\hat{a}_i(\tau) = \hat{r}_i(\tau+1) - \hat{r}_i(\tau). \tag{4.17}$$

To include the minimum on and off times we require that

$$\begin{aligned} \hat{r}_i(\tau) &= 0, \quad \tau = k, \ldots, k + T_{i,\text{off}} - \hat{t}_{i,\text{off}}(\tau), \\ \hat{r}_i(\tau) &= 1, \quad \tau = k, \ldots, k + T_{i,\text{on}} - \hat{t}_{i,\text{on}}(\tau) \end{aligned} \tag{4.18}$$

i.e. the μ-CHP is off for $T_{i,\text{off}}$ time-steps and on for $T_{i,\text{on}}$ time-steps. The dynamics of the on and off counters are specified by letting $\hat{t}_{i,\text{off}}(\tau+1)$ be incremented by one when $\hat{r}_i(\tau) = 0$ and reset to zero when $\hat{r}_i(\tau) = 1$. Similarly, $\hat{t}_{i,\text{on}}(\tau+1)$ is incremented by one with $\hat{r}_i(\tau) = 0$ and reset to zero when $\hat{r}_i(\tau) = 1$.

When the time constraints (4.18) are not active $\hat{r}_i(\tau)$ can take any value in the set $\{0,1\}$ and so allowing $\hat{p}_i(\tau)$ to take values in $P_i(\tau) = \{\hat{p}_i(\tau) | -p_{i,\max} \leq \hat{p}_i(\tau) \leq -p_{i,\min}\} \cup \{0\}$. This is modeled by the following constraint for all $\tau = k, \ldots, k + K_{\text{pred}}$,

$$\hat{r}_i(\tau) \cdot p_{i,\min} \leq \hat{p}_i(\tau) \leq \hat{r}_i(\tau) \cdot p_{i,\max}, \tag{4.19}$$

where K_{pred} is the prediction horizon.

A difference with *Problem QP* is that when we do find a solution, we are guaranteed that it is a solution that can be implemented in practice. However, as the problem is non-convex we will sometimes not converge in the iterations between the sub-gradient step for prices and the minimization to find a unique control input.

Remark 4.3.1. *Even though the price gradient iterations (4.11) converge for non-convex problems, the combined problem of gradient iterations and minimization of the local minimization might not converge when* $p_{i,\min} > 0$ *in (4.9).*

Choosing a Solution

We can expect the solution of the problem with the above constraints in some cases to oscillate between on and off. In the implementation we stop the sub-gradient-optimization iterations if the input starts to oscillate, and the on (off) state before the oscillation is implemented to the system. This way the constraints are met in the distributed MPC setting.

4.3.4 Results

Here we show the results from simulations combining our network model and constraints presented in Section 4.3.1 with the distributed MPC method reviewed in Section 2.3. We implement the constraints both as presented in Section 4.3.3.1 and in Section 4.3.3.2. Recall that the first method has the advantage that it is fast, while the second method has the advantage that all constraints are included explicitly in the decomposed optimization problems. In this section we take the agents in the network to be households. The solutions are found by a QP-solver and a MIQP-solver from GuRoBi version 4.6 [19] with python 2.5. For details about the implementation see [62].

Network Simulations

We perform simulations with the same realistic power demand patterns as in Section 4.2.2. There are 250 unique patterns representing five different types of households, and the demand patterns represent half a day in a November month. At this time of the year we can assume that the heat demand is high in the houses, in which case the heat production from the μ-CHPs is not wasted. The resolution of the demand patterns is one minute. For more details about the demand patterns see Appendix A.

Each house can have unique constraints on the capacity of the μ-CHP. However, in these network simulations all agents have the same production capacity. We are free to use any prediction model for the change in demand $\hat{w}_i(\tau)$, since the demand is an external signal, but an accurate forecast enables the controller to anticipate on the future behavior. For all simulations here, we assume that each household can exactly predict their change in demand patterns in the future, i.e. $\hat{w}(\tau) = w(\tau)$. Figure 4.4 - 4.6 are generated using a network with circular information topology, given by information matrix

$$A = \begin{bmatrix} 0.6 & 0.2 & 0 & 0 & 0.2 \\ 0.2 & 0.6 & 0.2 & 0 & 0 \\ 0 & 0.2 & 0.6 & 0.2 & 0 \\ 0 & 0 & 0.2 & 0.6 & 0.2 \\ 0.2 & 0 & 0 & 0.2 & 0.6 \end{bmatrix} \tag{4.20}$$

The difference in values compared to Figure 4.6, shows the design flexibility of the Information matrix. Each agent weighs its own imbalance with 0.6 and two neighbor imbalances with 0.2, i.e. the agents finds its own imbalance most important in this circular network.

Based on simulation results, we use a prediction horizon of $K_{\text{pred}} = 8$. It is a trade-off between computation-time and accuracy of the result. When the minimum off-time T_{off} (minimum on-time T_{on}) is shorter than the prediction horizon T, the MIQP solver becomes slow, as the combinatorial complexity rises. For all households $i = 1,\ldots,5$, the minimum production is $p_{i,\min} = 0.3$ kW and the maximum production is $p_{i,max} = 1$ kW, which are realistic values from a typical μ-CHP system. The minimum time off $T_{i,\text{off}} = 15$ min and the minimum on time $T_{\text{off}} = 15$ min are values abstracted from a μ-CHP present in our lab. We use gradient step size $\gamma_r = \frac{0.4}{r}$ in (4.11) at iteration number r, and the algorithm at one MPC time-step k terminates when $\Delta\lambda_i < 10$ for all $i = 1,\ldots,n$ ($\Delta\lambda_i < 50$ in Table 4.1).

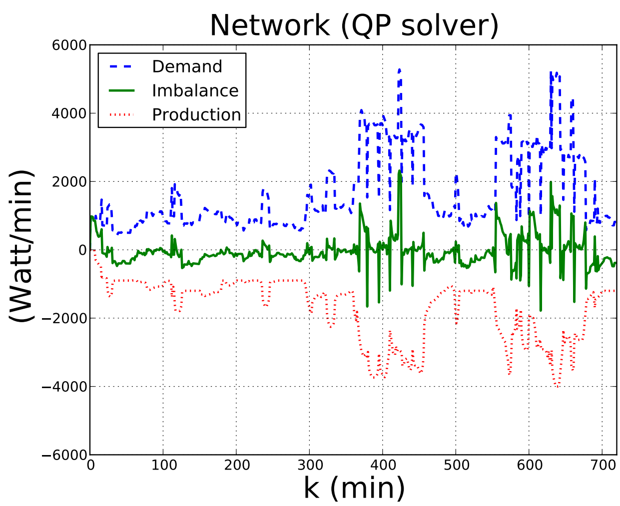
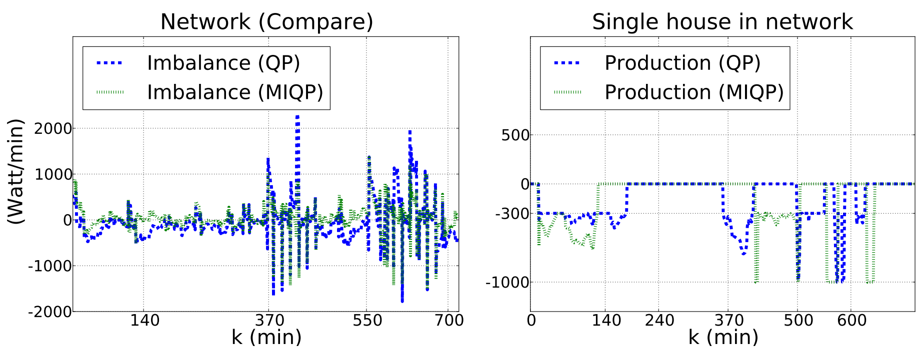


Figure 4.4 – The net imbalance $\sum_i x_i$, demand $\sum_i d_i$, and production $\sum_i p_i$ in a circular network of five households. The resolution is one minute, and the demand pattern for each household is unique.

Figure 4.4 shows the net imbalance, demand and production for the network of five distinct household types using the distributed version of *Problem QP*. We see that the power production, dotted line, mirrors the demand, dashed line, nicely. Consequently, the imbalance shown in the solid line, is steered towards zero. Because of the good prediction models for change in demand, the network can anticipate the future situation.

We notice in Figure 4.4, that the network is sometimes a net producer of power, see e.g. $k = 140$ where the imbalance is -475 kW. At other times the network is a net consumer, see e.g. $k = 370$ where the imbalance is 1152 kW. Without the μ-CHPs the network needs $7.20 \cdot 10^7$ kWh from the external line, and with the μ-CHPs the network needs $9.00 \cdot 10^5$ kWh from the external line. This is a reduction of 93.75 %. In addition, the network delivers $1.33 \cdot 10^7$ kWh in total in the same period.

For the network as a total, we obtain comparable results when we solve the distributed *Problem MIQP*. Figure 4.5(a) is included to explicitly compare the solutions from the two formulations. Here the dashed line shows the imbalance in the network with the QP solver and the dotted line shows the imbalance in the network with the MIQP solver. The solution found by the MIQP solver seems to regulate the state better to zero, see e.g. $k = 550$ in Figure 4.5(a) when a large change in the demand pattern has occurred.



(a) (b)

Figure 4.5 – (a) compares the net imbalance of the QP and the MIQP formulation. (b) compares the production for a single household in the network in the QP and MIQP formulation. Notice that the behavior at agent level differs in the two cases, but that both formulations steer the imbalance towards zero after a change in the demand.

There are clear differences at the household level. In Figure 4.5(b) we show the production patterns for one household, to see explicitly how a μ-CHP turns on and off. The dashed line shows the solution obtained from the QP solver and the dotted line shows the solution obtained from the MIQP solver. If we focus on the interval $k = 500, \ldots, 600$, we see that using the QP solver the μ-CHP turns off during k = 190,...,361 and k = 422,...,500. While using the MIQP solver, the μ-CHP is turned off in the time slot k = 141,...,420. This means that even if the overall network performs comparably in the two different formulations, the individual division of who is selling and who is buying from neighbors differs. This is natural since MPC is sub-optimal in nature, and for the QP solution we do not know the effect of turning on or off the μ-CHP as this notion is enforced outside the optimization problem. Notice however also from Figure 4.5(b) that when a μ-CHP is on, the production is modulated in the range 0.3 kW till 1kW in both cases, which shows that the constraints are satisfied.

The overall cost (4.4) in the network for the different implementations confirms the observation from Figure 4.5(a) that the network performs slightly better with the MIQP formulation than with the QP formulation. The centralized MIQP cost ($8.40 \cdot 10^7$) is 75% of the centralized QP cost ($1.15 \cdot 10^8$). This is expected because we do not know the on-off effect in the QP formulation. The distributed implementation has a higher cost in both cases. In the QP case the cost ($1.24 \cdot 10^8$) rises with 8.5% and in the MIQP the cost ($8.67 \cdot 10^7$) rises with 3.2%. This has two reasons. Firstly, the on-off behavior of the μ-CHP makes the problem non-convex, and secondly, the algorithm does not run until the price difference between two time steps is exactly zero.

Table 4.1 indicates that the distributed implementation scales well. For the distributed case we look at the average number of gradient iterations per node, while for the centralized case we look at computation time. The test is done for $n =$ 5, 50, 250 and 1000 households, and the value for the five households case is set to one. The computations per household rises from 1 till 3.38 for 5 compared to 250 households in the distributed case, and the ratio stays at 3.38 for 1000 households. The computation time rise from 1 till 27.27 for 5 compared to 250 households in the centralized case, but for 1000 households the centralized problem cannot be solved because the model was too large for the free GuRoBi license.

Prices

The prices vary over the network, depending on the local imbalance. Figure 4.6 shows the change in price for each household. We interpret negative values as prices higher than the equilibrium price and positive values as prices lower than the equilibrium price. The dotted line in Figure 4.6 is the price pattern corresponding to the dotted line in Figure 4.5(b). We observe that the value of the price in Figure 4.6

Table 4.1 – Indication of computation time, normalized to the five node case for the distributed and centralized implementation respectively. Numbers are from the QP-solver.

Number of nodes	5	50	250	1000
Distributed MPC	1	2.01	3.38	3.38
Centralized MPC	1	5.83	27.27	-

rises just before the μ-CHP is turned on at $k = 420$ in Figure 4.5(b). When the μ-CHP is switched on at $k = 420$ the value of the price immediately decreases. Since the price rises when power shortage rises, this stimulates the device to be turned on when needed. When the problem (4.10) is convex, the prices in the network are such that no-one benefits from producing more of less power. However, the price is not the price for power in a currency. In our model, we view the price as a weighing parameter for the distance to the equilibrium price for power. Indeed, when the network is in equilibrium, all agents will have a Lagrangian multiplier equal to zero.

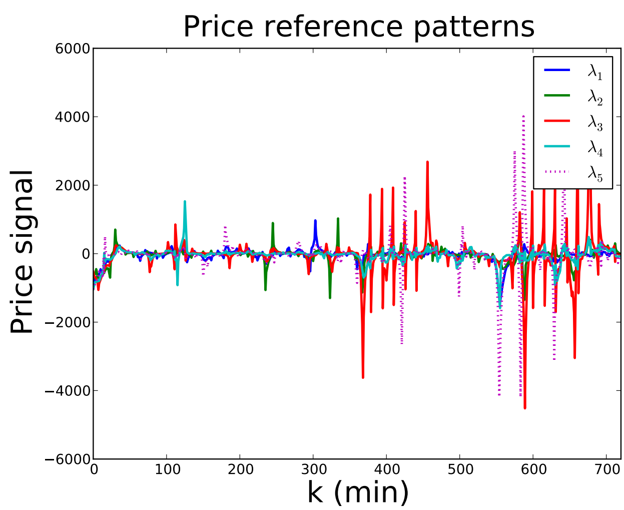


Figure 4.6 – The price references λ_i for each household type in the network.

4.3.5 Discussion

Here, we have shown that distributed MPC via dual-decomposition and sub-gradient iterations is a suitable design approach for embedding distributed generation in the power network at the end-user level. By coupling the agents dynamically through their notion of power imbalance information, and combining this model with the price mechanism, the network as a total converges fast to balance the power supply and demand.

The model has the freedom to take into account different generator capacity on each agent, and the agents may be weighed with different relative importance in the cost-function and in the information network.

We explicitly showed how to include μ-CHP systems with on-off behavior in the distributed MPC method. We include on-off behavior in two different versions in the simulations. One approach preserves the convexity in the optimization problem, in which case a fast algorithm can be used to find the solution and the theory reviewed in Section 2.3 is valid. The drawback is that the solution sometimes cannot tell if the μ-CHP is on or off. The other approach explicitly takes into account the on-off behavior, but we are faced with more slow mixed integer algorithms to find the solutions, and the problem is non-convex.

Based on the simulation results in Section 4.3.4, it can be concluded that both approaches steer the imbalance towards zero. The MIQP approach performs better with respect to the cost-function, while the QP approach is faster.

The current study has only examined flexibility in the power production, but when the price fluctuations are transparent for the end-user, we have reason to believe that we will also see flexibility in the demand [11]. We will discuss demand response in Chapter 6.

CHAPTER 5

Prosumers with μ-CHPs Including Heat Buffers

This chapter is based on our work presented in [40]. We extend the results of Chapter 4 and also consider the heat output from the μ-CHP systems. First, we present the model of a household with μ-CHP and heat storage. Then we combined it with the information sharing model developed in Chapter 3, and apply the distributed MPC algorithm presented in Chapter 2.

5.1 Problem Setting

In Chapter 1, we mentioned a demonstration project of a future energy-infrastructure in Groningen called PowerMatching city is implemented with 25 households[5]. The connected households have smart appliances, including μ-CHP systems that match their energy use in real-time based upon the available energy generation. Even though this demonstration project is generally perceived as a big success, it is also learning from some shortcomings, and thus improvements are considered. In particular, predictions of power and heat demand are not yet taken into account, as well as the coupling between heat and power, sometimes leading to a waste of heat production. This motivates to fully consider the coupling between heat and power, as well as predictions on the power demand.

The energy output of the μ-CHP can easily be controlled, even though it is subject to several operational constraints. In the literature, the μ-CHP unit is typically controlled according to heat-led or electricity-led strategies, meaning that the μ-CHP is turned on (off) according to the household's heat or power demand. However, in [20], it is stated that a combination of the two is most economically efficient. In Chapter 4, we considered the on-off and minimum on (off) times of the μ-CHP, and we coordinated the production in the network according to electricity-led control.

The primary goal of a μ-CHP, is however to cover the heat demand locally in the household. In this case, if the μ-CHP is controlled to follow the local heat demand, the electric power output fluctuates accordingly. Obviously, this may lead to a mismatch between local power supply and demand. However, by including storage in the network, we add flexibility to balance both heat and power. In particular, heat can be stored easily in water tanks in the household, which makes it easier to meet the heat demand. With the heat storage included, we can reverse the control problem in such a way that we control the power and store the heat. We then add the heat coverage problem as a constraint to the power control problem.

Electric power on the other hand, cannot be efficiently stored, but in the power network the power can easily be shared/traded amongst neighbors. This provides the opportunity to achieve a common goal at a network level such as matching local power supply and demand. The problem is however, how to coordinate the decisions made at household level regarding the μ-CHP in order to achieve such a power balance goal, and at the same time achieve heat balance.

Matching power in a large network, requires the control of a large number of agents. We therefore need a control strategy that scales well with the network, and we want the decision when to turn on or off a μ-CHP to be done by the household, only based on local information. In Chapter 4, the information sharing model presented in Chapter 3 was combined with the distributed Model Predictive Control (MPC) method reviewed in Chapter 2 to achieve power balance. A shortcoming in this previous work is that the heat output of the μ-CHP was not taken into consideration, meaning that when there is no heat demand the heat is wasted and the μ-CHP is not operated optimally from an energy efficiency and cost point of view. In reality the heat balance is a priority. In [23], Model Predictive Control has been applied to operate a single μ-CHP with heat-storage for cost minimization. In this chapter, we will focus on optimizing the integrated heat and power matching in a network of households with μ-CHP and heat storage. Therefore, we extend the analysis of the previous chapter to include heat demand and dynamics from heat the storage at each household. This results in a completely local heat supply and demand balancing, and at the same time the power supply and demand is balanced in the network based on local decisions. We compare the total amount of resources used in the network, with and without μ-CHPs present.

5.2 System Description and Problem Formulation

Here we first describe the agent model, being a household with μ-CHP and heat storage. This model specifies the technical constraints at each agent. Then we introduce the information sharing network of such households, which will help to achieve a balance of power in the network.

5.2.1 The Household

The setup of an agent is shown in Figure 5.1. The agent has a μ-CHP fuelled on natural gas, and the power output is connected to the power network while the heat output is stored in a hot water storage present in the house. The μ-CHP consists of a prime mover whose power and heat output is coupled, and an auxiliary burner which only has a heat output. Here we choose the prime mover to be a PEMFC[1], because of the high electric efficiency of this technology, see Table 5.1. This means that we have high flexibility to control the power output and at the same time stay within given heat constraints. The auxiliary burner produces heat like a conventional boiler, and also stores the heat in the hot water storage. The auxiliary burner is part of the control problem because if the power demand is low and the heat demand is high, we prefer that the heat demand is covered by the auxiliary burner. In other words, we want to avoid a net power production in the network due to high heat demand. Further, the heat storage has to meet the households demand of hot water and central heating.

The μ-CHP can be modeled with different degrees of technical detail. Here we are interested in a model capturing the main features of the μ-CHP suitable for control of the electric power and heat output. Such a model of a prime mover was presented in [23] and [22] for demand response in one household. We use the same model for the μ-CHPs. Notice that this is a different μ-CHP model than the one considered in Chapter 4.

Each agent i has an electric power demand $d_i(k) \in \mathbb{R}_+$, and a heat demand $h_{d,i}(k) \in \mathbb{R}_+$ that can be measured at each discrete time-step k, where the heat demand is the aggregated space heating and domestic hot water needs. The prime mover produces electric power $p_i(k) \in \mathbb{R}_-$ with an efficiency $\eta_{p,i}$ and heat $h_{p,i}(k) \in \mathbb{R}_+$ with an efficiency $\eta_{h,i}$ when it is on. The efficiencies are approximated by a constant over the energy output, even if the efficiency can be slightly higher for a

[1]Proton Exchange Membrane fuel cell

prime mover at low loads. The power and heat output from the prime mover are therefore coupled to the power output by

$$h_{c,i}(k) = -\eta_i p_i(k), \tag{5.1}$$

where $\eta_i = \eta_{h,i}/\eta_{p,i}$, and the minus sign is due to our choice that the power production $p_i(k)$ is opposite in sign to the power demand $d_i(k)$. There is a minimum and maximum power output $p_{\min,i}, p_{\max,i} \in \mathbb{R}_+$ that the μ-CHP can deliver. These constraints are given by

$$p_i(k) \in \{0\} \ \cup \ [-p_{\min,i}, -p_{\max,i}]. \tag{5.2}$$

The auxiliary burner provides additional heat $h_{a,i}(k) \in \mathbb{R}_+$ at times of high heat demand, and its range is given by

$$h_{a,i}(k) \in \{0\} \ \cup \ [h_{a,\min,i}, h_{a,\max,i}]. \tag{5.3}$$

This heat production has an efficiency $\eta_{a,i}$. The non-convexity due to the gap between 0 and $p_{\min,i}$, and 0 and $h_{a,\min,i}$, has consequences for the control algorithms that can be applied.

We assume that there are no thermal losses in the conversion and storage system, therefore, the dynamics of the heat storage level $h_{s,i}(k) \in \mathbb{R}_+$ is given by

$$h_{s,i}(k+1) = h_{s,i}(k) + h_{p,i}(k) + h_{a,i}(k) - h_{d,i}(k), \tag{5.4}$$

as in [23].

The maximum and minimum heat storage levels $h_{\min,i}, h_{\max,i} \in \mathbb{R}_+$ of the hot water tank depend on the mass m of the water, the specific heat constant c_p and the difference in minimum (maximum) temperature of the water and the room-temperature $\Delta T_{\min\,(\max),i}$, and is given by

$$h_{\min\,(\max),i} = m_i c_p \Delta T_{\min\,(\max),i}, \tag{5.5}$$

where c_p = 4.18 kJ/(kg·K). We assume that no heat can be dumped, and the heat storage level needs to stay within the limits

$$h_{\min,i} \leq h_{s,i}(k) \leq h_{\max,i}, \tag{5.6}$$

at all times k.

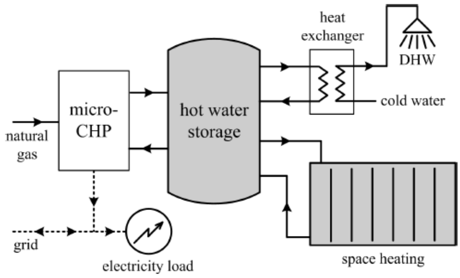


Figure 5.1 – This Figure is taken from [21], and shows the energy flow in one household.

At last, the prime mover has a relatively long start-up time $T_{\text{start},i}$ when it has been shut down. During this period an amount $g_{r,i}$ of gas is consumed, but no electricity and a neglectable amount of heat are produced. If the prime mover is frequently shut down, this could limit the opportunity to control the power output. A choice can therefore be to keep the prime mover at an idle on-state, in which state the prime mover can resume production at any time. However, in an idle on-state the prime mover consumes the minimum gas $g_{i,r}$ required to keep the reforming unit at the operating temperature, even if no power and heat are produced.

5.2.2 The Information Network

The agent's operation of the μ-CHP will influence the power balance in the power network. We propose an information sharing model in a network of n agents connected through an information network, which does not need to have the same structure as the corresponding power network. This will also help us to implement a completely distributed control strategy in the network.

According to Chapter 3, we introduce a virtual information network, so that each agent has local information about the system when making the decision. The topology of the information network specifies which subset of agents an agent i exchanges information with. Agent i's set of information neighbors N_i is given by

$$N_i \subseteq \{1,\dots,n\} \backslash \{i\}, \tag{5.7}$$

where the agent itself is excluded.

We include the chosen information topology in our dynamic model by adjusting information weights $A_{ij},\ i,j = 1,\dots,n$ in the coupling between the agents' notion of imbalance in the system. The difference between power production $p_i(k)$ and demand $d_i(k)$ at an agent i is the *imbalance information* $x_i(k) \in \mathbb{R}$ at agent i, and the model for the imbalance information is given by

$$x_i(k+1) = A_{ii}x_i(k) + \sum_{j \in N_i} A_{ij}x_j(k) + u_i(k) + w_i(k), \tag{5.8}$$

for all $i = 1, \ldots, n$, where $u_i(k) = p_i(k+1) - p_i(k)$ is the change in power production, $w_i(k) = d_i(k) - d_i(k-1)$ is the change in power demand, A_{ii} weighs the power imbalance information of agent i itself, and A_{ij} weighs the information received from neighbors $j \in N_i$. We choose the initial value of $x_i(0)$ to be the real physical power imbalance of agent i at the initial time, i.e. $x_i(0) = d_i(0) + p_i(0)$ and $w_i(0) = 0$.

5.2.3 Central Model Predictive Control of the Network

Due to the many operational constraints from the μ-CHP and heat buffer presented in Section 5.2.1, we will solve the optimal control problem using MPC, which means that we optimize a control problem with finite horizon K_{pred} at each time step k, see e.g. [8]. The network goal will be expressed by means of an objective function, and local forecast about future energy demand can be taken into account when the control actions are determined. We also include prediction models based on the agents' and network dynamics and constraints presented in Section 5.2.1 and Section 5.2.2. This means that we anticipate on how each agents' change in power demand and production affects the network power balance over the horizon, while explicitly taking into account the heat and operational constraints from the μ-CHP. New measurements of power demand, heat demand and heat storage levels are included as initial conditions at each MPC cycle. This corrects for inaccuracies in the models, and energy forecasts.

In order to include the non-convex constraints (5.2) and (5.3) in our MPC problem, we introduce binary decision variables $r_i(k)$ and $r_{a,i}(k)$ as in [23]. These variables are defined by

$$\begin{aligned} r_i(k) &= \begin{cases} 1 & \text{if the } \mu\text{-CHP is on,} \\ 0 & \text{otherwise,} \end{cases} \\ r_{a,i}(k) &= \begin{cases} 1 & \text{if the auxiliary burner is on,} \\ 0 & \text{otherwise,} \end{cases} \end{aligned} \tag{5.9}$$

and allow us to replace (5.2) and (5.3) by

$$\begin{aligned} -p_{\min,i} \cdot r_i(k) \geq \quad & p_i(k) \geq \quad -p_{\max,i} \cdot r_i(k), \\ h_{a,\min,i} \cdot r_{a,i}(k) \leq \quad & h_{a,i}(k) \leq \quad h_{a,\max,i} \cdot r_{a,i}(k). \end{aligned} \tag{5.10}$$

In that case that the μ-CHP is completely off before starting up again, as described in the end of Section 5.2.1, we also need to include the start-up time $T_{\text{start},i}$ when the μ-CHP is on, but does not produce power or heat. We introduce two new binary variables to indicate whether the primary mover is still in the start-up phase, or is shut down at a given time-step k

$$\begin{aligned} a_{up,i}(k) &= \begin{cases} 1 & \text{if primary mover starts up at } k, \\ 0 & \text{otherwise,} \end{cases} \\ a_{down,i}(k) &= \begin{cases} 1 & \text{if primary mover shuts down at } k, \\ 0 & \text{otherwise,} \end{cases} \end{aligned} \tag{5.11}$$

with relations

$$\begin{aligned} r_i(k) - r_i(k-1) &= a_{up,i}(k) + a_{down,i}(k), \\ a_{up,i}(k) + a_{down,i}(k) &\leq 1, \\ r_i(k+p) &\geq a_{up,i}(k), \;\; p = 0, \ldots t_{\text{start},i}(k), \end{aligned} \tag{5.12}$$

where $t_{\text{start},i}(k)$ is a counter that is decreased by 1 each time-step k after the μ-CHP was turned on, and $t_{\text{start},i}(k)$ is reset to $T_{\text{start},i}$ when $a_{down,i}(k) = 0$. Relations (5.12) ensure proper behavior of the μ-CHP.

As we have seen, the agent has different choices for how to cover its energy need. It can get power from the network, or from its μ-CHP, and it can get heat from the primary mover or the auxiliary burner. We therefor specify the objective function, which will specify the control goal. The global control goal is to minimize the overall power imbalance in the network, while keeping the appropriate temperature in the local heat storage. For this purpose we define the local objective function

$$V_i(x_i(k), u_i(k), h_{a,i}(k)) = Q_{ii} x_i^2(k) + R_{ii} u_i^2(k) + H_{ii} \Delta h_{a,i}^2(k), \tag{5.13}$$

where $Q_{ii}, R_{ii}, H_{ii} > 0$ weigh the relative importance of the imbalance information, change in power production and change in auxiliary heat output respectively. In addition to minimizing the imbalance and change in power production, we also included the heat output of the auxiliary burner in the objective. This is done to make sure the auxiliary burner is only on when it is needed to meet the heat storage constraints, and we choose $H_{ii} << Q_{ii}$. We use hat-notation to indicate predictions of the variables presented in Section 5.2, and minimize the following global objective

$$\sum_{\tau=k}^{k+K_{\text{pred}}} \sum_{i=1}^{n} V_i(\hat{x}_i(\tau), \hat{u}_i(\tau), \hat{h}_{a,i}(\tau)), \tag{5.14}$$

subject to the prediction models of (5.1), (5.4), (5.6), (5.8), (5.9), (5.10), (5.11), and (5.12) for all $i = 1, \ldots, n$, and $\tau = k, \ldots, k + K_{\text{pred}}$.

Solving (5.14) requires one central controller that has full information about states, measurements and forecasts of all agents in the network. However, the decision to turn on or off the primary and auxiliary burner in such a centralized way does not scale well [39]. The decisions will instead be taken completely distributed only based on local information, as presented next.

5.2.4 Distributed Model Predictive Control of the Network

The minimization (5.14) can be approximated by dual-decomposition and subgradient iterations, as explained in Section 2.3. The main idea is that the neighbor influence in (5.8), is replaced by a local guess $v_i(k)$ constrained by

$$v_i(k) = \sum_{j \in N_i} A_{ij} x_j(k), \tag{5.15}$$

which decomposes the state equation (5.8)

$$x_i(k+1) = A_{ii} x_i(k) + v_i(k) + u_i(k) + w_i(k). \tag{5.16}$$

Then the constraints (5.15) are moved to the objective function by standard Lagrangian relaxation, which introduces the Lagrangian multipliers $\lambda_i(k)$ related to the constraints (5.15). The complete decomposition is achieved by including subgradient iterations.

In Algorithm 2, the distributed MPC method is sketched. Given a sequence $\{\hat{\lambda}_i(\tau)\}_{\tau=k}^{k+K_{\text{pred}}}$, each agent $i = 1, \cdots, n$ solves a local control problem

$$\text{minimize } V_{i,k,K_{\text{pred}}}, \tag{5.17}$$

subject to prediction models of (5.1), (5.4), (5.6), (5.16), (5.9), (5.10), (5.11), and (5.12) for all $\tau = k, \ldots, k + K_{\text{pred}}$, in order to find $\{\hat{p}_i(\tau)\}_{\tau=k}^{k+K_{\text{pred}}}$ and $\{\hat{h}_{a,i}(\tau)\}_{\tau=k}^{k+K_{\text{pred}}}$. The objective is given by

$$V_{i,k,K_{\text{pred}}} = \sum_{\tau=k}^{k+K_{\text{pred}}} \Big(V_i(\hat{x}_i(\tau), \hat{u}_i(\tau), \hat{h}_{a,i}(\tau)) + \hat{\lambda}_i(\tau)\hat{v}_i(\tau) - \sum_{j \in N_i} \hat{\lambda}_j(\tau) A_{ji} \hat{x}_i(\tau) \Big), \tag{5.18}$$

where $V_i(\hat{x}_i(\tau), \hat{u}_i(\tau)$ is defined in (5.13). The solution of (5.17) is found using a mixed integer quadratic program, see [23],[3]. Then $\{\hat{\lambda}_i(\tau)\}_{\tau=k}^{k+K_{\text{pred}}}$ is updated according to

$$\hat{\lambda}_{i,r+1}(\tau) = \hat{\lambda}_{i,r}(\tau) + \gamma_{i,r}[\hat{v}_{i,r}(\tau) - \sum_{j \neq i} A_{ij} \hat{x}_{j,r}(\tau)], \tag{5.19}$$

where r labels the sub-gradient iteration and $\gamma_{i,r}$ is the gradient step size. The Lagrangian multipliers are often viewed as price signals. The algorithm terminates when the update of the Lagrangian multipliers stays within a bound ε.

For convex problems the solution of Algorithm 2 converges to the solution of the centralized problem (5.14). However, due to the non-convex constraint (5.2) and (5.3), modeled by including binary variables (5.9), such a convergence cannot be guaranteed. In fact, we expect situations to occur where the binary variables (5.9) oscillate between zero and one. This means that the update of the Lagrangian multiplier, at each iteration r, changes the solution of the optimization problem from $r_i(k) = 1$ to $r_i(k) = 0$ and back. In this situation, it is not clear what is the optimal state of the μ-CHP. We deal with this problem by fixing the binary variable as explained in Remark 5.2.1.

Remark 5.2.1. *In order to use Algorithm 2 for our non-convex problem, we will fix the binary inputs (5.9) once a non-converging sequence of (5.17) and (2.51) is detected. The remaining overall optimization problem will then become convex, since the gap between zero and the minimum value in (5.2) or (5.3) is removed, and the values of the other variables will converge to values that are optimal given the fixed binary values.*

Result: Find $u_i(k), h_{p,i}(k), h_{a,i}(k)$ at each cycle k of the distributed MPC method

for $k = 0, \ldots, K_{sim}$ **do**
  each agent i measures $p_i(k), x_i(k), h_{s,i}(k), w_i(k)$;
  while $|\hat{\lambda}_{i,r}(\tau) - \hat{\lambda}_{i,r-1(\tau)}| > \varepsilon$ **do**
    for $i = 1, \ldots, n$ **do**
      solve (5.17);
    end
    each agent i exchanges $\hat{x}_i(\tau)$ to connected agents;
    for $i = 1, \ldots, n$ **do**
      sub-gradient update (2.51);
    end
    each agent i exchanges $\hat{\lambda}_i(\tau)$ to connected agents;
  end
  each agent i implements
  $u_i(k) = \hat{u}_i(\tau)|_{\tau=k}, h_{p,i}(k) = \hat{h}_{p,i}(\tau)|_{\tau=k}, h_{a,i}(k) = \hat{h}_{a,i}(\tau)|_{\tau=k}$;
end

Algorithm 2: Distributed Model Predictive Control, and K_{sim} is the simulation time

5.3 Results

Here we use the distributed MPC algorithm, as described in Sections 5.2.3 and 5.2.4, to control the network of households with μ-CHP and heat storage, described in Section 5.2. We compare the solution of Algorithm 2 to the centralized solution (5.14).

The implementation is done in Python with a mixed integer solver from the Gurobi optimization library, see [19]. For the distributed implementation we set the step size in (2.51) to be $\gamma_{i,r} = \frac{0.01}{\sqrt{r}}$, where r is the iteration number. This might not be the optimal strategy of updating the step size in order to converge fast. However, it suits to prove the concept of the distributed implementation. The algorithm is terminated when all distances between the previous and current Lagrangian multiplier satisfy $|\hat{\lambda}_{i,r}(\tau) - \hat{\lambda}_{i,r-1(\tau)}| < 0.003$ for all $i = 1, \ldots, n$ and $\tau = k, \ldots, k + K_{\text{pred}}$. Other possible ways to terminate the algorithm could be to choose a fixed number of sub-gradient iterations r, or specify a criterion for the desired degree of sub-optimality of the solution compared to the centralized solution as presented in Chapter 2.

Since we use the algorithm which is valid for convex problems for a integer problem, we need to implement Remark 5.2.1 in order for the algorithm to always converge and terminate. Therefore, when we detect that a state $\hat{r}_i(\tau)$ switches six times from the on state to the off state during the sub-gradient iteration, the binary variables $\hat{r}_i(\tau)$ are fixed to their latest value. When an integer is fixed it means that the new problem is changed compared to the original problem.

In Figure 5.3-5.5 we have used a circular topology. All agents weigh themselves by a weight 0.8, and two neighbors by a weight 0.1 in the information matrix A. The simulation is done for $K_{sim} = 720$ minutes with a resolution of 1 minute, the prediction horizon is $K_{\text{pred}} = 5$ minutes, and we assume that each household can predict its demand patterns exactly over these five time-steps. This means that $\hat{w}_i(k) = w_i(k)$, $\hat{w}_i(\tau) = 0$ for $\tau = k+1, \dots, k+K_{\text{pred}}$, and $\hat{h}_{d,i}(\tau) = h_{d,i}(k)$ for $\tau = k, \dots, k+K_{\text{pred}}$. We use realistic power demand and corresponding heat demand patterns [49]. The power demand patterns are the same as in Chapter 4. The patterns are from a November month when the heat demand is high, and the resolution is one minute. Each household has unique patterns based on five different household profiles. Figure 5.2 shows the demand patterns for a single household with a low energy demand profile, and for 25 households, which is the size of the PowerMatching city mentioned in the introduction to this chapter. We see that the power demand has fluctuations of the order 1 kW, the space heating demand is smooth and of the order 2.5 kW for the large households, while the hot tap water demand has short spikes with demand as high as 20 kW. Covering the tap water demand is only possible when using the auxiliary burner, even with the presence of heat storage. All patterns show a peak around $k = 250$ minutes, and for the heating patterns there is also a smaller peak around $k = 550$ minutes.

The parameters for the μ-CHP and heat storage model presented in Section 5.2.1 are given in Table 5.1. Notice that we here model a different μ-CHP unit than in Chapters 4-5. Here the power output is in the range 0.1-3.0 kW instead of 0.3-1.0 kW. The primary mover produces 70% heat and 30% power, and the auxiliary burner produce 100% heat. At the beginning of the simulation all μ-CHPs are turned off, $p_i(k) = h_{p,i}(k) = h_{a,i}(k) = 0$ kW, and the temperature of the heat storages are $60°C$.

We compare the distributed and central solution of the following four scenarios;

Case 1 No μ-CHPs are installed and the electricity demand is imported to the network, while the heat demand is covered by boilers with 100% efficiency

Case 2 All households have a μ-CHP, but the auxiliary burner is not considered

Case 3 As Case 2, but only every third house has a μ-CHP and the households without a μ-CHP can get power from their neighbors and heat from a boiler

Case 4 All households have a μ-CHP and the auxiliary burners are considered in the control problem.

Table 5.2 and Table 5.3 report the total resources used for generating imported power, local generated power, heating the reforming unit and producing local heat in the network, total running time of the optimization per time-step k, average number of sub-gradient iterations r per time-step k, and the objective V given by

$$V = \sum_{k=1}^{K_{sim}} \sum_{i=1}^{n} [x_i^2(k) + u_i^2(k)]. \tag{5.20}$$

The gas consumption of the prime mover $g_p(k)$ and the auxiliary burner $g_a(k)$ are modeled by

$$g_{p,i}(k) = -p_i(k)/\eta_{p,i}, \tag{5.21}$$

$$g_{a,i}(k) = h_{a,i}(k)/\eta_{a,i}. \tag{5.22}$$

In order to calculate the resources used to generate the imported power from external parties an efficiency of 45% is assumed.

Table 5.2 shows the results for 10 households. In Case 1, the total amount of resources needed to cover the net energy demand is 351 kWh. In Case 2, the total amount of resources needed to cover the same energy demand is increased by 9%. The reason for this can be seen in Figure 5.3. The network produces too much power, because the μ-CHP has to operate at full capacity due to the large heat demand. Notice that excess production is particularly high in the period $k = 200 - 400$ minutes, which is correlated with the peak in the heat demand in Figure 5.2. In fact, the lower heat storage constraint is even violated around 90 minutes per household. Figure 5.6(a) shows household 10 in the network as an example of this. The central and distributed algorithm performs comparable with respect to the use of resources, the objective is twice as high for the distributed algorithm, but it is about 10 times faster than the centralized solution. Table 5.3

shows that when the network increases to 25 households the distributed algorithm is 30 times faster than the centralized solution for Case 2.

The same observations with respect to resources, objective and running time hold for Case 3. Here every third household has a μ-CHP, and the rest of the households can buy power from their neighbors and produce heat with a conventional boiler. Figure 5.4 shows that the network can cover its power demand in this case, but the lower heat storage level is violated for the households with a μ-CHP during 40 minutes for each of these households. Figure 5.6(b) shows household 10 in the network as an example of this. However, Table 5.2 shows that almost 9% is saved on the resources.

Figure 5.5 shows that in Case 4, when all households have a μ-CHP with an auxiliary burner and heat storage, the power production can be well matched with the power demand in the network. At the same time, the heat demand is covered at all time. This can be confirmed by Figures A.4(a) - A.6(d) in Appendix A. The central solution shows that we can achieve a maximum of 15% saving in the resources compared to not having μ-CHPs in the network. The distributed solution has a 7% saving compared to Case 1, but we can expect that this can be improved by tuning the parameters in the sub-gradient iterations. In Case 4 the number of binary variables are doubled, i.e. both $r_i(k)$ and $r_{a,i}(k)$. This has consequenses for the scalability of the centralised implementation. Table 5.3 shows that for 25 households, no solution could be obtained within 1 minute for the central implementation, while the distributed implementation still finds a solution within 0.003 seconds for Case 4.

Table 5.1 – Parameters for the μ-CHP and water storage

Parameter	Value	Unit
η_p, η_h	0.3, 0.7	-
$p_{\min}$, $p_{\max}$	0.1, 3.0	kW
p_{ramp}	0.15	kW/min
T_{start}	45	min
g_r	1	kW
η_a	1	-
$h_{a,\min}$, $h_{a,\max}$	4.0, 20.0	kW
m	200	l
$T_{\min}$, $T_{\max}$	55, 80	$^{\circ}C$

Table 5.2 – Simulation results for half a day, $n = 10$.

Case	Method	V	Resources	Time	Iterations
1	-	-	351 kWh	-	-
2	Cent	1.30 kWh^2	384 kWh	0.022 s	-
	Dist	2.34 kWh^2	385 kWh	0.002 s	14.7
3	Cent	0.09 kWh^2	320 kWh	0.013 s	-
	Dist	1.03 kWh^2	330 kWh	0.001 s	9.8
4	Cent	0.02 kWh^2	296 kWh	0.474 s	-
	Dist	0.22 kWh^2	328 kWh	0.012 s	22.2

Table 5.3 – Simulation results for half a day, $n = 25$.

Case	Method	V	Resources	Time	Iterations
1	-	-	936 kWh	-	-
2	Cent	3.28 kWh^2	983 kWh	0.063 s	-
	Dist	6.04 kWh^2	975 kWh	0.002 s	31.8
3	Cent	0.38 kWh^2	842 kWh	0.031 s	-
	Dist	2.73 kWh^2	840 kWh	0.001 s	21.6
4	Cent	-	-	-	-
	Dist	0.74 kWh^2	857 kWh	0.003 s	27.1

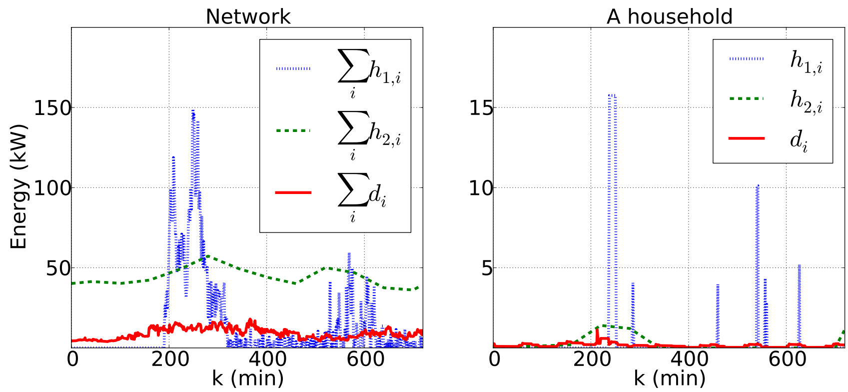


Figure 5.2 – The hot tap water $h_1(k)$, space heating $h_2(k)$, and electric power demand $d(k)$ of one household (right) and the sum of 25 households (left). In particular the tap water demand has large spikes distributed over the day. The aggregated plot shows that there is a peak in energy demand around $k = 250$ minutes.

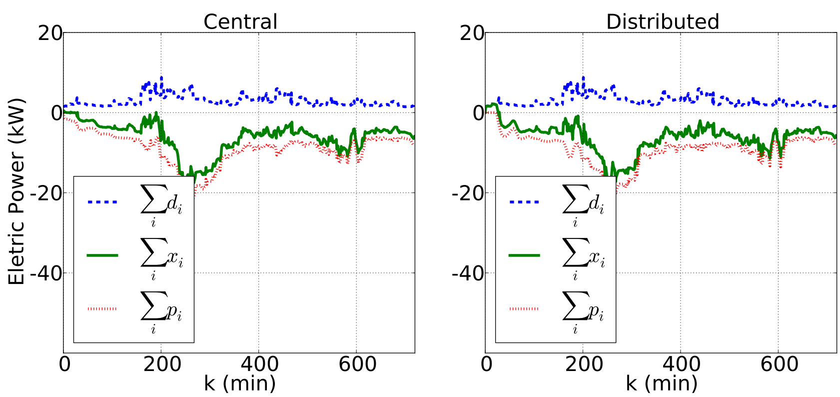


Figure 5.3 – Net electric power demand $\sum_i d_i(k)$, imbalance $\sum_i x_i(k)$, and production $\sum_i p_i(k)$ in the network of 10 households for Case 2, where all households have a μ-CHP but the auxiliary burner is not included. There is a large excess of power production in the network, in particular between $k = 200$ and $k = 400$ minutes, when the heat storage levels hit its lower boundary and the μ-CHP is forced to produce at full capacity.

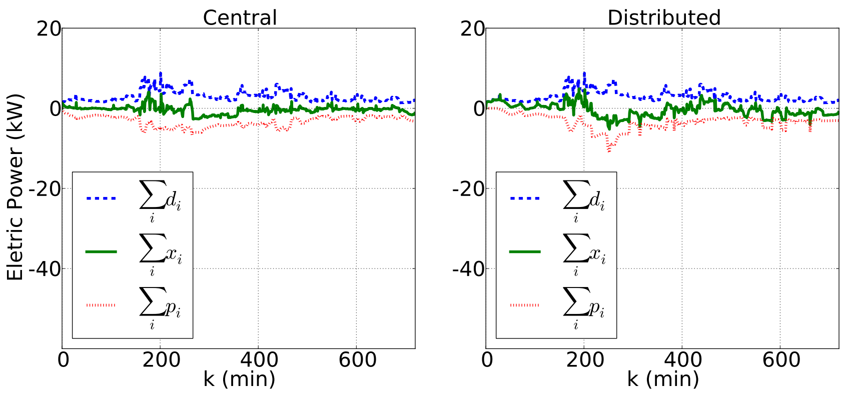


Figure 5.4 – Net electric power demand $\sum_i d_i(k)$, imbalance $\sum_i x_i(k)$, and production $\sum_i p_i(k)$ in the network of 10 households for Case 3, where the difference from Figure 5.3 only every third house has a μ-CHP. We see that there is now enough generated power to cover the demand in the network. However, the μ-CHP cannot provide enough heat for the local consumption. On average, the heat storage level lower boundary is violated 40 minutes per household with a μ-CHP.

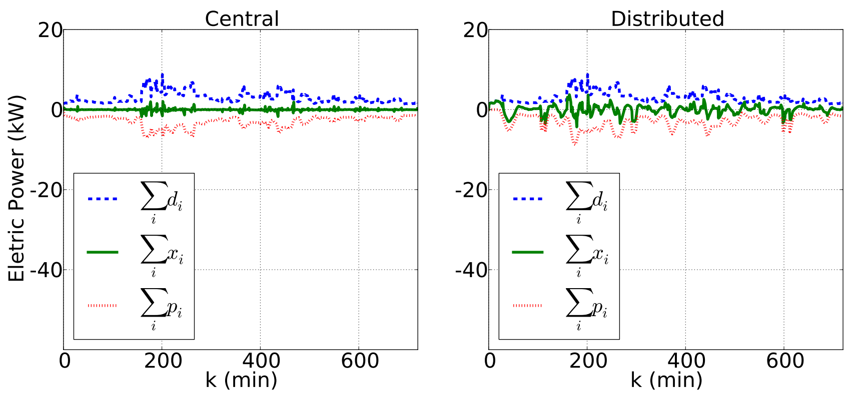
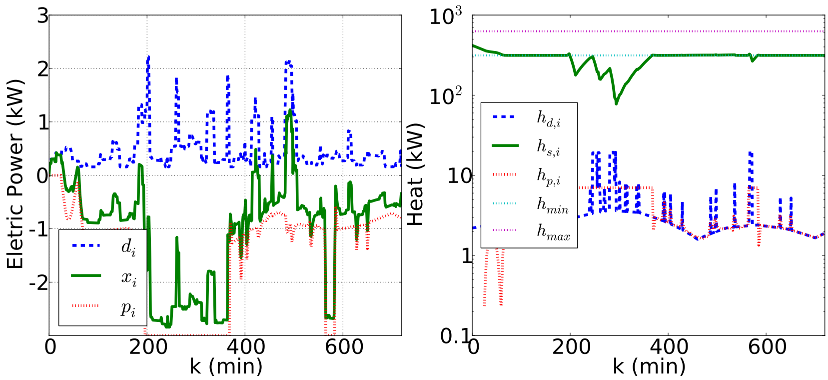
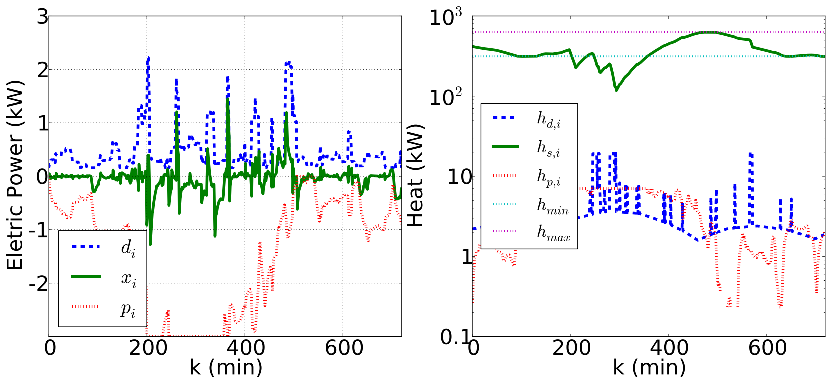


Figure 5.5 – Net electric power demand $\sum_i d_i(k)$, imbalance $\sum_i x_i(k)$, and production $\sum_i p_i(k)$ in the network of 10 households for Case 4, where all households have a μ-CHP and the auxiliary burner is included. We see in the left plot that the network can balance the net power production and consumption, and at the same time the heat storage levels stay within its boundary at all times. By tuning the step-size and stopping criterion of the sub-gradient iterations, the distributed implementation shown in the right plot can also perform better.



(a) Case 2, agent 10



(b) Case 3, agent 10

Figure 5.6 – Here (a) zooms in on agent number 10 in the network corresponding to Figure 5.3. We see that at k = 200 the heat storage constraint is violated even if the μ-CHP is operating on full power. Because there is not a high power demand in the network, the imbalance is negative. (b) zooms in on agent number 10 in the network corresponding Figure 5.4. In this case, at k = 200 the heat storage constraint is still violated, but the electric power output from the μ-CHP is used by neighbors in the network. Therefore, the imbalance is closer to zero in (b) than in (a).

5.4 Discussion

In this chapter, we modeled a network of households with μ-CHPs and heat storage. The model has the freedom to include different production capacities and heat storage sizes at each household. By including information sharing dynamics, we achieve a balance between the power supply and demand in the network at the same time as the heat demand at household level is satisfied. The decision making is done by each household, and the decisions are coordinated by sub-gradient iterations as described in the distributed MPC algorithm presented in Section 5.2.4. The algorithm is guaranteed to converge to the centralized solution for convex problems. However, the output from the μ-CHP and auxiliary burner are non-convex, meaning that our control problem is not convex. Still, based on the simulations, when we implement Remark 5.2.1 in the algorithm, the distributed solution is comparable to the centralized solution. We compare the distributed and the centralized implementation, and study the influence on the resources needed to match supply and demand.

The distributed method can be implemented in a large network, since the number of optimization variables per node is constant. Increasing the network, however, slowly increases the number of sub-gradient iterations needed to match the stopping criterion. However, recall that in Chapter 4 the same number of sub-gradient iterations was needed for networks of 250 and 1000 households. The centralized problem cannot be solved for large problems, in particular due to the large number of binary variables. Already with 25 households we could not solve Case 4.

Based on the simulation results, we conclude that when embedding the μ-CHPs and heat storage in the network, the network as a total uses less resources than when no μ-CHPs are present, and the auxiliary burner is needed to stay within the heat limits. The distributed control approach is suitable, as it scales well and helps balance the network.

We considered a circular information topology, while we could have considered any topology as long as the restrictions for the information matrix A given in Section 3.2 are met. How the steady state solution and the transient response of the system are influenced by the A matrix, is not considered in this chapter. However, in the case that not all households have a μ-CHP, the households with a generator should be distributed evenly over the information network, since the information distance between two households determines how fast information travels from one node to the other.

The current study has examined distributed coordination of power production in a network with μ-CHPs and heat storage elements. However, when the price fluctuations are transparent for the end-user, we have reasons to believe that we will also see flexibility in the demand, see e.g. [11]. In the next chapter the modeling of the power demand side together with the distributed control approach is studied.

The current study assumes that the information network is a subset of the power network. The goal is to match power in the network at a market level, so that the power exchange with the overall network is minimized. For further study it is also interesting to include power balance equations, so that the information network could operate as a stand alone network.

CHAPTER 6

Control of a Network with Multiple Electricity Consumers

This chapter is based on our work presented in [35]. Individual users in the Smart Grid can also contribute to optimize the system by the means of demand response [31], not just by the means of distributed generation as treated in Chapters 4-5. This means that the demand of the individual users in the network will be balanced dynamically and continuously to match the supply (or a time with lower demand) of electric power. In other words, we aim to control the demand flexible demand side devices in such a way that the net power load in the network is flattened. Benefits from a flatter demand are discussed in Chapter 1, i.e. the thesis introduction.

Domestic devices with flexibility to participate in demand response are typically devices such as refrigerators, since the temperature can be controlled to stay within a certain temperature range, and the washing machine program can be shifted in time. When the number of households gets large, this flexibility can help to drive the power demand in the grid towards a target value when the decisions for when to turn on are coordinated. The resulting optimization problem that would need to be solved at a central computing center has a high combinatorial complexity.

We model a network with washing machines whose switch-on time has to be coordinated at two levels. Firstly, the switch-on time has to be coordinated with the overall power demand in the system. Secondly, the switch-on time has to be coordinated with the expected switch-on time of neighbors in the network. Further, a user will first try to flatten his own and virtually connected neighbors power demand. Later, information from more far away neighbors in the information network will also be considered.

We take the approach of the distributed MPC method presented in Chapter 2 combined with the information sharing model presented in Chapter 3. In particular, we show how distributed MPC can be used to coordinate washing machines in a local power network. Our model with the distributed MPC method is tested with realistic power demand patterns from typical Dutch households. Due to the binary nature of the constraints a Mixed Integer Quadratic Program is used to find the solution of the local problem, given price signals from information neighbors. We check scalability of the problem, in terms of computation time per household, and we advise an algorithm implementation with dynamic allocation of the households in an alternative case with distributed computation centers instead of computers present at each household.

6.1 Heavy Demand Consumers in a Network

The system consists of a network of households that make decisions for when to turn on the washing machines and exchange information according to a specified topology. Here, the input $u_i(k)$ will be completely determined after deciding the binary on-off state of the washing machine. Notice that compared to the notation in Chapter 3, we here use $d_{f,i}(k) = f_i(k)$, $d_{e,i}(k) = g_i(k)$, $p_{f,i}(k) = 0$, and $p_{e,i}(k) = 0$.

6.1.1 Network Model

We model a network of $n \in \mathbb{N}$ households, where each house $i = 1, \dots, n$ has an average electric power demand $d_i(k) \in \mathbb{R}_+$ measured over each discrete time-interval k. This demand consists of a shiftable power demand $f_i(k) \in \mathbb{R}_+$ that is a result of the decisions the household makes, and a non-shiftable part $g_i(k) \in \mathbb{R}_+$ that is modeled as an external signal. The total demand of house i at time-step k is therefore given by

$$d_i(k) = f_i(k) + g_i(k), \tag{6.1}$$

where $f_i(k)$ is associated with the demand from washing machines in this chapter.

We repeat (3.5) here

$$x_i(k+1) = A_{ii}x_i(k) + \sum_{j \in N_i} A_{ij}x_j(k) + u_i(k) + w_i(k), \tag{6.2}$$

where the state $x_i(k)$ of household i is this household's information about demand which will be shared with the neighbors. As before, it is a combination of the household's own and its information neighbors' demand, where the weights are specified by the information matrix A. The information matrix has to meet requirements R1-R4 given in Section 3.2.1. Further, $u_i(k) = f_i(k+1) - f_i(k)$ is the change in electric power demand from the washing machine at household i, and $w_i(k) = g_i(k+1) - g_i(k)$ is the change in the rest of electric power demand at household i. In this way, when household i makes its decision for when to turn on the washing machine, it is not only taking its own demand into consideration, but also the situation in the network it is a part of. As a result, coordination of decisions in the network is enabled.

6.1.2 Network Cost

The goal is to make the power demand of the network more flat. This means that we want to move shiftable demand away from peaks in the demand pattern. The objective function is therefore given by

$$\sum_{\tau=k}^{k+K_{\text{pred}}} \sum_{i=1}^{n} [\hat{x}_i(\tau) - a]^2, \tag{6.3}$$

which will be minimized when the individual demand information $\hat{x}_i(\tau)$ is close to a target value a. Here a could for example be the average power demand in the network divided by n. Notice the difference from (3.12). We have omitted the term that is quadratic in $u_i(k)$, because the value of $u_i(k)$ is completely determined once the starting time of the wash has been determined.

6.1.3 The Washing Machine Model

Once a washing program is turned on, the washing machine has to remain on until the program is finished. The washing machine can only turn on if the machine is loaded, and it has to finish the program before a specified time $T_{i,\text{f}} \in \mathbb{N}_+$. This parameter together with the number of time-steps it takes for the program to finish $T_{i,\text{p}} \in \mathbb{N}_+$ is specified when the washing machine is loaded.

The washing machine at house i is loaded $n_i \in \mathbb{N}_+$ number of times per day, according to an external signal $\eta_{i,\text{load}}(k)$ which is 1 if it is loaded at time k and zero otherwise. The washing machine can only be re-loaded if it has finished the previous program, and the arrival time is random. Binary variables $\delta_i(k)$ and $\mu_i(k)$ specify whether the washing machine is running

$$\delta_i(k) = \begin{cases} 1 & \text{if washing machine is running at time } k, \\ 0 & \text{otherwise,} \end{cases} \tag{6.4}$$

and whether it is loaded

$$\mu_i(k) = \begin{cases} 1 & \text{if the washing machine is loaded at time } k, \\ 0 & \text{otherwise.} \end{cases} \tag{6.5}$$

which differs from $\eta_{i,\text{load}}(k)$ that is only nonzero at one time-step. In other words, $\eta_{i,\text{load}}(k)$ represents the action of being loaded. Therefore, $\mu_i(k) = \mu_i(k-1) + \eta_{i,\text{load}}(k)$ unless $\delta_i(k-1) = 1 \bigwedge \delta_i(k) = 0$ when $\mu_i(k)$ is reset to zero. Another constraint is that the washing machine can only run if it is loaded, i.e. $\delta_i(k) := 0$ if $\mu_i(k) = 0$.

Next we need to ensure that the washing machine completes the program within the time $T_{i,\text{p}}$ after it has started. We introduce $t_{i,\text{on}}(k) \in \mathbb{N}_+$ to count the number of time-steps the washing machine has been running. The counter is incremented with one when $\delta_i(k) = 1$ and set to zero when $\delta_i(k) = 0$. The resulting constraints on $\delta_i(k)$ are therefore

$$\delta_i(k) = \begin{cases} 1 & \text{if } \delta_i(k-1) = 1 \bigwedge t_{i,\text{on}}(k-1) < T_{i,\text{p}} \\ 0 & \text{if } t_{i,\text{on}}(k-1) = T_{i,\text{p}} \end{cases} \tag{6.6}$$

It is also a requirement that the washing machine is finished before the given finish-time $T_{i,\text{f}}$, and for the implementation we introduce another counter $t_{i,\text{w}}(k)$ that counts the number of time-steps the machine has waited from when it was loaded. This means that it is incremented with one when $\mu_i(k) = 1$ and $\delta_i(k) = 0$, and reset to zero when $\delta_i(k) = 1$. The washing machine is forced to start running $\delta_i(k) := 1$ when the maximum waiting time is reached $T_{i,\text{p}} - t_{i,\text{on}}(k) \leq T_{i,\text{f}} - t_{i,\text{w}}(k)$.

Finally, the change in shiftable demand $u_i(k) = f_i(k+1) - f_i(k)$ has to get the right value when the program is running. The demand follows a pattern $e_i(t_{i,\text{on}}(k)) \in \mathbb{R}_+$ when it is on. This means that

$$f_i(k) = \begin{cases} e(t_{i,\text{on}}(k)) & \text{if } \delta_i(k) = 1 \\ 0 & \text{if } \delta_i(k) = 0 \end{cases} \tag{6.7}$$

We will use a realistic power demand pattern $e(t_{i,\text{on}}(k))$ for the washing machine, see Figure 6.1.

6.1.4 Distributed MPC Problem

The task is to choose when to turn on the washing machine so that the decoupled problems

$$\begin{aligned} \underset{\hat{u}_i, \hat{v}_i}{\text{minimize}} \quad & \sum_{\tau=k}^{k+K_{\text{pred}}} \Big([\hat{x}_i(\tau) - a]^2 + \hat{\lambda}_i(\tau)\hat{v}_i(\tau) - \sum_{j\neq i} \hat{\lambda}_j(\tau) A_{ji} \hat{x}_i(\tau) \Big), \\ \text{subject to} \quad & (2.43) \text{ and the washing machine model in Section 6.1.3,} \end{aligned} \tag{6.8}$$

are solved together with the price iterations

$$\hat{\lambda}_{i,r+1}(\tau) = \hat{\lambda}_{i,r}(\tau) + \gamma_{i,r}[\hat{v}_{i,r}(\tau) - \sum_{j\neq i} A_{ij}\hat{x}_{j,r}(\tau)], \tag{6.9}$$

which means determining $u_i(k) = \hat{u}(\tau)|_{\tau=k}$ given the network model and the operational constraints from the washing machine present at the household.

6.2 Results

Here we show how the computations can be distributed over a computer network, we perform a simulation with realistic demand pattern, and look at the scalability of the algorithm.

6.2.1 Implementation

The hardware we use for the results presented in this section are four dual xeon-processor servers with a total of 32 cores. More specifically Intel(R) Xeon(R) CPU L5420 at 2.50GHz.

We implement the network model in Python 2.7, and use a mixed integer quadratic program from the Gurobi library [19] to find the solutions of the optimization problems. To parallelize the optimizations from each household, we used mpi4py [26], [27]. The MPI interface provide an essential communication functionality between a set of processes. Typically, for maximum performance, each CPU (or core in a multi-core machine) will be assigned just a single process. In our case, each core represents one household.

In practice the communication follows a master-slave model. The structure of the implementation is a master plus computational slaves with one household mapped to one computational slave. The computation goes in steps with complete synchronization between steps, i.e. all computations are finished before the next time-step. See Subsection 6.2.3 for improvements on this scheme.

The IF and AND operators in the model presented in Section 6.1.3, such as (6.6) and (6.7), are included by rewriting the equations in terms of several inequality constraints. See [50], [3] for details. In (6.7), auxiliary variables are introduced to indicate at which time-step in the washing program we are, so that the right value of the demand pattern $e(t_{i,\mathrm{on}}(k))$ can be picked.

With the binary on-off decisions, the problem is no longer convex as the theory in Section 2.3 requires. We can expect a situation to occur when the binary decision variable oscillates between on and off. When such a situation occurs, we will fix the binary variable so that the sub-gradient iterations can converge to the optimal solution given the fixed binary variable. This is equivalent to Remark 5.2.1.

6.2.2 Simulations

We use realistic power demand patterns [49] provided by the Energy research Center of the Netherlands (ECN). Each household has a unique demand pattern based on five typical household profiles of a November evening. The resolution of the simulations are seven minutes.

The information network consists of $n = 20$ households exchanging information according to

$$A = \begin{bmatrix} 0.6 & 0.2 & 0 & 0 & \cdots & 0.2 \\ 0.2 & 0.6 & 0.2 & 0 & \cdots & 0 \\ 0 & 0.2 & 0.6 & 0.2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & 0.2 & 0.6 & 0.2 \\ 0.2 & \cdots & 0 & 0 & 0.2 & 0.6 \end{bmatrix}, \tag{6.10}$$

which means that each households has two information neighbors. They weigh themselves with a weight 0.6 and two neighbors with a weight 0.2.

The power demand patterns of the washing machine is obtained from consumption patterns provided by the Flexines project [25]. Flexines measured the electricity consumption per minute for 12 different programs of the Whirlpool Texas 1400 washing machine. In the simulations presented here, a cotton wash program at 40 ^oC is chosen. This demand pattern $e(t_{i,\mathrm{on}})$ is shown in Figure 6.1.

We perform the simulations for one evening in November month, and assume that 10 of the 20 households have one wash to be done this evening, and we randomly load the washing machines during the first 2 hours and 20 minutes of the evening. As a benchmark case, the households then immediately turn on the machine, see Figure 6.2. The green solid line is the total demand in the network, the blue dashed line is the unshiftable demand and the red dotted line is the shiftable demand from the washing machines. We call this the uncontrolled case, and we

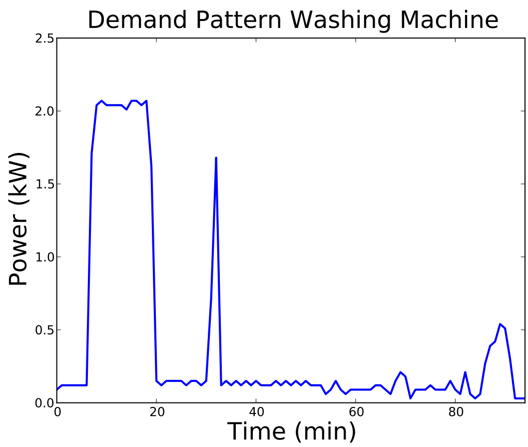


Figure 6.1 – Demand pattern of a 40 ^oC cotton program.

will compare the cost

$$\sum_{k=0}^{K_{\mathrm{sim}}} \sum_{i=1}^{n} [x_i(k) - a]^2, \tag{6.11}$$

to the situation where the d-MPC scheme is used to determine when to start the machines. The resolution of the simulation is 7 minutes, the simulation time K_{sim} is 7 hours, the prediction horizon K_{pred} in (6.8) is 4 hours and 40 minutes, and after the machine is loaded it must finish before $T_{i,\mathrm{f}}$ = 4 hours and 40 minutes. The $\gamma_{i,r}$ in (6.9) is 0.001 and the stopping criterion for the sub-gradient iterations is $|\hat{\lambda}_{i,r}(\tau) - \hat{\lambda}_{i,r-1}(\tau)| < 0.05$ for all $i = 1, \ldots, 20$ and all $\tau = k, \ldots, k + K_{\mathrm{pred}}$. The target value for each household is chosen to be approximately the average load per household over the simulation, i.e. $a = 500$ Watt. See the blue dashed line in Figure 6.2.

Figure 6.3 shows the result when the d-MPC scheme presented in Section 2.3.2 is implemented. Compared to Figure 6.2 where all washing machines turn on during the first 2 hours and 20 minutes, the loads from the washing machines are now spread out over the two periods where the overall demand in the network is lowest, i.e. $k = 0 - 120$ minutes and $k = 220 - 320$ minutes. In fact, when the unshiftable demand is high, then the demand from the washing machines is low, and thus the washing machines are shifted to the moment that the unshiftable demand is somewhat lower. This is also reflected in the cost (6.11). In the benchmark case the cost was 248.56 kWatt^2, while in the d-MPC case the cost was 231.61 kWatt^2. All programs are finished before the specified finish-time.

Figure 6.4 shows one of the households in the network with a relative high unshiftable demand, see the blue dashed line. The demand information, the green solid line, illustrates how the household share information with neighbors. From $k = 140$ minutes the demand information decreases even though the unshiftable demand and the shiftable demand of the household stay fairly constant. This information sharing with neighbors is the reason why the demand information is negative at $k = 210$ minutes, when the unshiftable demand decreases. The washing machine turns on shortly after at $k = 230$ minutes.

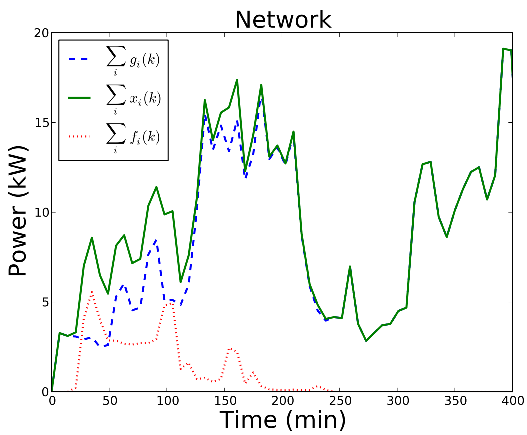


Figure 6.2 – The benchmark case; an uncontrolled network. Here 10 of the 20 households are loading a wash at a random time during the first 2 hours and 20 minutes. This load is shown in the dotted line, while the load that can not be controlled is shown in the dashed line. We notice that some of the shiftable load is located at 150 minutes when the unshiftable load is high.

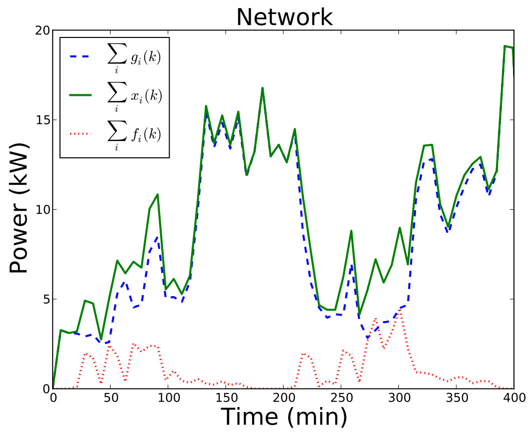


Figure 6.3 – With communication and d-MPC algorithm. Compared to Figure 6.2, we see that the demand from the washing machines has been shifted to points in time where the overall power demand is low.

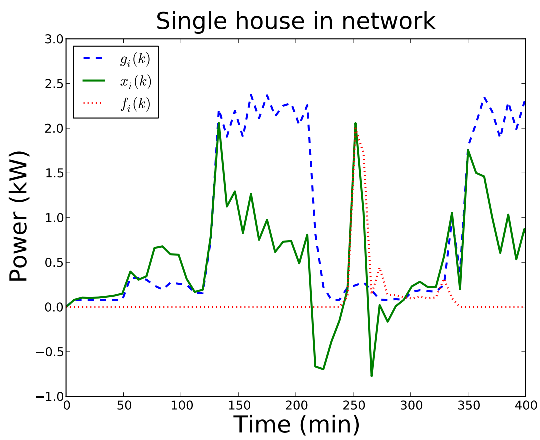


Figure 6.4 – Zooms in on one of the households in the network shown in Figure 6.3. A household of the type five persons in a stand alone house.

In the simulation presented above, the average computation-time for one time-step k was 6 seconds and the maximum computation-time for one time-step k was 15 seconds. This is well within the simulation time resolution of 7 minutes. Using the centralized setup, it was not possible to solve the problem using the software and hardware we used.

For scalability it is important to know that the computation time is not exploding. To check the scalability of the d-MPC optimization, the average computation-time per time-step k, and the average number of sub-gradient iterations per optimization per time-step k are given in Table 6.1. All n households are loaded with one wash. Notice the difference from Figures 6.2-6.4 where only half of the network is loading a wash.

Table 6.1 – Scalability of the d-MPC problem.

n	8	16	32
Average computation time per time step k in seconds	5.7	5.9	6.4
Average # of iterations	32.7	33.5	37.1

6.2.3 Scalability of the Algorithms

In Subsections 6.2.1 and 6.2.2, we assume that the household network and the computational network are the same. Because each household has a similar computational requirement, this solution scales quite well. However, the situation can be more flexible, with for example geographically scattered small computation centers across the network or only some of the households also being a computational nodes, whereas some others are clustered on a node. The only requirement is that the number of computational cores in total is high enough in order for a time step k in the d-MPC scheme to finish within the time resolution of the system.

The computational load of each household varies a lot, i.e. it is high when the washing machine is loaded and low when there is no shiftable demand. With the implementation in the previous subsection, the computation-time for a household with a loaded washing machine was approximately 50 times longer than a household without shiftable demand.

In the case where the available processors in the network are limited, it is important to make optimal use of the available processors. We therefore need to make the mapping of households to cores dynamic. Concretely the mapping can change each cycle in the computation. This means that the computation for a household sometimes has to migrate from one core to the next. In this migration we only have to migrate the execution segment [60] of the program, because the code segment is the same everywhere.

In a large network, the relative number of nodes involved in migration is relatively small, and because the size of the execution segment is not large (here approximately 4KB), the process migration adds relatively little to the overall computational load of the system. Also, using multi-cores, the master can make the mapping so that most migration communication is local to processors.

Each household can predict when a period of high computational load arrives, since a high computational load means that a washing machine is loaded. This has to be communicated to the master which can then change a mapping and initiate the process migrations.

6.3 Discussion

The results in this chapter show that d-MPC via dual-decomposition and sub-gradient iterations is a suitable design approach for embedding demand response in the smart grid. By exchanging price information with a few neighbors in the network, the turn-on-time of washing machines in the network was coordinated with the overall demand in the network. Due to the distributed nature of the approach, the problem is expected to scale well as the number of households get large. This was also seen in Section 6.2.2 where the scalability of the problem was tested.

The simulations show that the turn-on time of the washing machines has been coordinated on two levels. Firstly, the flexible demand has been shifted to the time of lower demand in the overall demand pattern. Secondly, the turn-on time of the washing machines are distributed so that not all machines turn on at once. This is in line with what we required from the algorithm.

The theory of Chapter 2 guarantees that the solution for the centralized and the distributed approaches are the same for convex problems. However, the non convexity of the on-off constraints changes the nature of the problem. We successfully included these constraints, resulting in a network with lower costs than when no demand side control is implemented. This also accords with our earlier observations in Chapter 4 when using the approach to embed distributed generation in the power network.

A further study with focus on a larger variation of devices suitable for demand response, such as freezers, present at the households is suggested. It also has to be connected to local production devices. Another interesting topic is further development of the scalability considerations for the algorithms as discussed in Subsection 6.2.3.

CHAPTER 7

Conclusion and Outlook

This dissertation has investigated distributed control in a multiple electricity prosumers network, where the aim is to reach a global goal in the network only based on distributed decisions and information. In this chapter, we discuss the results and main contributions of this thesis, and we finish with recommendations for further work.

7.1 Conclusions

This project was undertaken to design an infrastructure for the smart grid that allows the agents in the grid to apply distributed control so that the network as a total reaches a global goal for electricity production and consumption. We also evaluate the distributed control algorithms based on dual-decomposition and sub-gradient iterations using the infrastructure, by the means of case studies with supply-side and demand-side appliances.

In Chapter 3, we have designed *an information sharing model* for agents with power production and consumption in a network. This model serves as a corner stone for the case studies in this thesis, and it enables distributed coordination of electric power in the network. In the model, the agents are dynamically coupled through their power imbalance information. The agents do not have full information about the power imbalance in the complete network, but they communicate imbalance information to a few connected neighbors.

We have distinguished between a virtual information network where the agents exchange information, and the physical power grid where the agents are connected. The agents exchange information with a subset of the agents in the information network, and the net mismatch of power production and consumption in the information network is imported (exported) to the overall power network. This exchange has not been explicitly modeled, but the objective can be formulated so that this exchange is minimized, or steered to a target value. It is also important to notice that we work on a market level, which is the same viewpoint as the PowerMatcher mentioned in Chapter 1 and further details can be found in [33]. This means that we have designed a model with the interest in the amount of power that is produced and consumed, and the physics of the power network itself has not been explicitly included. Nonetheless, this is a useful approach since we can choose the virtual neighbors such that the agents trade energy in their close neighborhood in the physical power grid. For example, when an agent's virtual neighbors are connected in the same low voltage network, the load at the corresponding transformer station can be kept more flat.

Further, we have provided four requirements for how the agents weigh information from information neighbors. These requirements were mainly motivated by stability and conservation of information considerations. However, even when meeting the four requirements the information matrix still has design flexibility with respect to network structure and the value of the weights. We have given an example of how the information matrix could be built for the current electricity network, and we have argued that we can choose for a rather sparse information structure in order to facilitate the scalability.

In Chapter 4, we have considered *coordination of decentralized power production*. We have seen in a case study on μ-CHPs in a network that decentralized power production can be controlled in a distributed manner to match the power consumption in the network only by considering local information.

First, we have considered a network with agents that have discrete-time linear time-invariant dynamics, with all variables taking on continuous values. Simulations with realistic data have shown that by applying the dual-decomposition method reviewed in Chapter 2 to the information sharing model in Chapter 3, a balance between power production and demand can be achieved only based on local decisions and information. Stochastic peaks are present in the imbalance plots, but these peaks will be less present in a large network. However, since the control inputs have not been constrained, this first simulation only serves as a proof of concept. Letting $u(k)$ be free would imply that we need for example battery storage and demand planning in time, because a μ-CHP has a maximum and minimum production and start-up (shut-down) dynamics. Still, the input is never completely free. Nevertheless, we have concluded that for a five households network a one kW electric μ-CHP system per household is able to produce a sufficient amount of electricity most of the dayin the given scenario.

Second, we have included predictions (forecast), on-off constraints, and a minimum on (off) time to the μ-CHP models. We have applied the distributed MPC method from Chapter 2 to incorporate predictions and constraints. However, the on-off characteristics of the μ-CHP system introduce challenges for the control method, which is only valid for convex problems. The distributed MPC method from Chapter 2 cannot be applied straightforwardly due to the binary variables introduced to the model. We have implemented two different algorithms to deal with this issue. First, we have implemented a quadratic programming (QP) algorithm where we let all variables be continuous in the optimization problems. If a solution is found that is between zero and the minimum power output the μ-CHP can deliver, the value that is closest is implemented. Next, we have implemented a mixed integer quadratic programming (MIQP) algorithm, where the binary nature of the problem is explicitly included in the optimization problem. In this case, it sometimes happens that the update of the Lagrangian multiplier at each sub-gradient iteration changes the solution of the optimization problem from turn-on to turn-off and back. When we detect such an oscillation between the on and off state, we choose to fix the binary input where the problem occurs. Then the sub-problem including this variable is convex. The simulations have shown that both the QP and the MIQP algorithm steer the imbalance to zero. We have seen that the MIQP approach performs better with respect to the objective function, while the QP approach is faster. This is natural since the MIQP algorithm takes the effect of the on-off behavior explicitly into account, but by doing so the introduced mixed integers results in a complex combinatorial problem.

In Chapter 5, we have included the heat output from the μ-CHP in our model. We have used a setup with households in a network including μ-CHP units and heat buffers. The μ-CHPs primary goal is to cover the heat demand locally in the household. In this case, if the μ-CHP is controlled to follow the local heat demand, the electric power output fluctuates accordingly. This results in a mismatch between local power supply and demand. However, by including heat storage in the network, we add flexibility to balance both heat and power. In particular, heat can be stored easily in water tanks in the household, which makes it easier to meet the heat demand. With the heat storage included, we have reversed the control problem in such a way that we control the power and store the heat. We have then added the heat coverage problem as a constraint to the power control problem.

This scenario-study has shown that compared to the current resource usage in the network assuming 100% efficient boilers and a 45% efficient power plant, the 3 kW μ-CHP with heat storage can cover both the local heat and power demand with less overall use of resources. However, when each household has a water tank of 200 liters for heat storage and a μ-CHP to cover all the heat, i.e. excluding an auxiliary burner, the power output from the μ-CHPs exceeds the total power demand in the network. This is because the μ-CHPs are forced to turn on to cover the heat demand, which is large compared to the electricity demand. The network is then a net exporter of electric power. Given the particular μ-CHP model and the Dutch power and heat demand patterns in late autumn, we have suggested that in late autumn only every third house has a μ-CHP system installed in order to achieve power balance in a network with only households for almost all the time. In this case, even less resources such as gas, coal or wind are used to produce the energy to cover the energy demand in the information network. In the case that all households have a three kW μ-CHP with an auxiliary burner, all heat and power demand in the network are covered locally at all time. Most of the time the power output is then less than 1 kW per household, which suggests that not all households have to invest in a three kW μ-CHP in order to cover the local power demand.

In Chapter 6, we have seen that distributed MPC via dual-decomposition and sub-gradient iterations is a suitable design approach for embedding *demand response* in the smart grid. By exchanging shadow price information with a few neighbors in the network, the turn-on-time of washing machines in the network was coordinated with the overall demand in the network.

The empirical findings in this study have shown that the turn-on time of the washing machines has been coordinated on two levels. Firstly, the flexible demand has been shifted to the time of less demand in the overall demand pattern. Secondly, the turn-on time of the washing machines are distributed so that not all machines turn on at once. Hence, the overall power demand is flattened.

We have also considered the *scalability of the algorithms.* Taken together, the results in this thesis suggest that the distributed approach scales better than the centralized approach. For example, in Chapter 4, the distributed case uses 3.38 times longer to solve for 250 households compared to 5 households. The centralized approach, on the other hand, used 27.27 times longer to solve 250 households compared to 5 households. In a large network, it is not even possible to solve a model with binary variables with the available software. In the most complex setting in Chapter 6, only 10 households could be included in the GuRoBi optimizing tool in the centralized case, while the distributed method in principle has no such limitations.

The findings in this report are subject to at least three limitations. First, only one type of electricity production and one type of demand device was modeled. Second, we only considered a static circular topology. Third, we did not perform experiments in the lab with real devices. In the next section we provide a further discussion on how to handle these and other limitations.

7.2 Future Work

This research will serve as a foundation for future studies. In a current project between the University of Groningen and DNV KEMA, the information sharing model with distributed MPC is incorporated in the Universal Smart Energy Framework (USEF). This is a framework developed by the Smart Energy Collective, which is a consortium of companies related to the energy world. The framework provides designs, specifications, and implementation guidelines for a smart energy system. More information about USEF can be found in [10].

This thesis has provided a method where flexibility of power supply and demand at the power prosumers can be coordinated to reach a common goal by means of a scalable algorithm. However, the research has raised many practical questions in need of further investigation. Some of the important practical implications are that the markets, policies, and power grid itself all have to be upgraded in order to facilitate two way flow of electric power.

In the current market, several types of stake-holders are responsible for, or are exploring business opportunities at various levels in the power system. The power producers sell the power at the highest possible price to the balance responsible parties, who again sell the power to power suppliers, consumers or other balance responsible parties. Small end-users buy the power from suppliers, and can be contracted by a range of energy companies in the free market. In this market the prices for power and the prices for the infrastructure are separated. There are also regulated parties involved in the power system. The Transmission System Operator (TSO) is responsible for the transmission grid and the real-time balance of the power supply and demand. The TSO ensures the safety of the power grid and thus supervises the energy transactions. Next to the TSO, we have the Distribution System Operator (DSO) which is responsible for the low voltage and medium voltage distribution grids.

One question is then, how do we organize the information sharing network, i.e. design the information matrix introduced in Chapter 3, with respect to these existing market parties? Should the information sharing network be formed by the free market, optimized with respect to technical performance, or should each energy company, who today contracts the small consumers, form independent information sharing networks? One way to organize it is to let the DSO be responsible for the information sharing network. This way, the flexibility can be coordinated to benefit the distribution network. The DSO can pay the end-users for the flexibility they can provide in the information network. Then the DSO is free to set a target value for the power balance at which the network will operate. This target value might even change with time. The distributed algorithm is then used to coordinate the controllable devices to reach this target value.

In case the DSO is in charge of designing the information matrix, a related question is then, how does the DSO best design the information matrix to achieve the target power value as quickly as possible? In this thesis, we only considered a circular topology with static weights. However, we can imagine that the available flexibility varies from prosumer to prosumer, and that the flexibility also varies over a day per prosumer. It is best for the network if a prosumer with high flexibility receives imbalance information as fast as possible. Therefore, prosumers with a low flexibility should inform a prosumer with high flexibility, and thus, these two prosumers should be close information neighbors.

It is interesting to assess the effects of different information network topologies. The best information topology might even be time varying if the flexibility changes over time in the network. On one hand we should consider the effect of different topologies on the convergence speed of the algorithm. It is clear that the structure and weights in the information matrix will affect both the steady state solution and the transient response of the system. On the other hand we need to consider what is the best information topology given a power grid topology. By choosing information neighbors in a reasonable way, transportation losses can be minimized. In addition, we can keep the load over the transformers close to a target value. This is beneficial for the network configuration. It is cheaper when there can be more connections on each transformer, and grid reinforcements can be delayed or prevented. Taken together, it would be interesting to look for conditions for an optimal information matrix (A matrix) with respect to quantities like network losses, congestion over the transformer and convergence speed of the algorithm.

In Chapter 5, we conclude that at late autumn only every third house needs a specific μ-CHP model installed to cover the local heat and power demand. Natural questions that relate to the discussion in the previous paragraph are then the following: Where do we place the μ-CHPs, and how do the participants in the information network best split the economical benefit? Regarding the μ-CHP placement with respect to the information network, we expect that the μ-CHPs should be spatially distributed so that the average information distance from an agent to an agent with a μ-CHP is as small as possible. This is because the information about changes in demand then reaches the agent with production faster. The placement with respect to the power grid is a separate but related question, but we expect that at least every low voltage network connected to one LV-MV transformer should have enough capacity to cover the local demand most of the time. This is to minimize the transportation losses.

A further study with focus on a larger variation of electrical devices suitable for demand response is also suggested. This has to be connected to local production devices and possibly including storage devices. Moreover, the method has to be included in a field test to establish whether real devices in households can be coordinated efficiently with respect to the network goal. The method should also be compared to heuristic methods from practice.

In this thesis we only considered a network goal to balance power demand and production. Other types of objective functions can also be explored as goals in the network. One might incorporate environmental goals, transportation losses, and economical costs. In fact, with an economic cost function the Lagrangian multipliers may get a translation to a currency such that users can get a fair bill for their electric power as pointed out in Section 2.1.2. In Chapter 4, we interpret the shadow prices as the distance to the equilibrium price in the network. However, it is an open question how to bill each member in the information network in practice. In [38], we show the shadow price patterns produced by the methods researched in this thesis for a two-agent network. When there is a step in the demand of one of the agents the price rises faster here than at the neighbor. When the network reaches a balance, the integral of the shadow price flattens. This behavior depends on the weights in the information matrix. In a large network, when there is a large change in demand or supply at one agent, the shadow price rises most at this agent and neighboring agents. If the imbalance is high then the shadow price rises quicker than if the imbalance is low. When there is a balance between production and demand in the network, the shadow prices do not change. The shadow prices will be highest at the prosumer where the imbalance occurred, and at his close information neighbors. This means that close by information neighbors will have the opportunity to react fastest to an imbalance. That the close by information neighbor has the highest shadow price also means that he has the most to earn by producing. Thus, price signals spread as a ripple on the water through the connections in the information network.

With respect to the scalability of the distributed algorithm, we saw in Table 4.1 that the distributed algorithm in this case study needed the same number of iteration in the 1000-agent network as in the 250-agent network to reach the stopping criterion. It would be interesting to investigate at what network size a further increase in the number of agents, no longer results in an increase of the number of sub-gradient iterations. Also, how sensitive is this network size to quantities like the size of a change in external power, type (size) of stopping criterion, and amount of flexible power present in the network?

Other aspects that are not touched upon in this thesis are the social, psychological, privacy, and regulation aspects of a future grid. We leave this to the experts of the respective fields.

APPENDIX A

Complementary Data

We use realistic power demand patterns [49] provided by the Energy research Center of the Netherlands (ECN). The demand patterns is from November month 2011, and the resolution is one minute. Each of the households in the networks in Chapters 4 - 6 has a unique demand pattern, based on the following five different profiles:

1. Single person in an apartment with an aware user profile. No mechanical ventilation.
2. Two persons in a semi-detached house, with an average user profile. No mechanical ventilation.
3. Two persons in a sixties terraced house, with an average user profile. No mechanical ventilation.
4. Three persons in a modernized sixties terraced house, with a cost saving user profile. Mechanical ventilation present.
5. Five persons in a stand-alone house, with a comfort oriented user profile. Mechanical ventilation present.

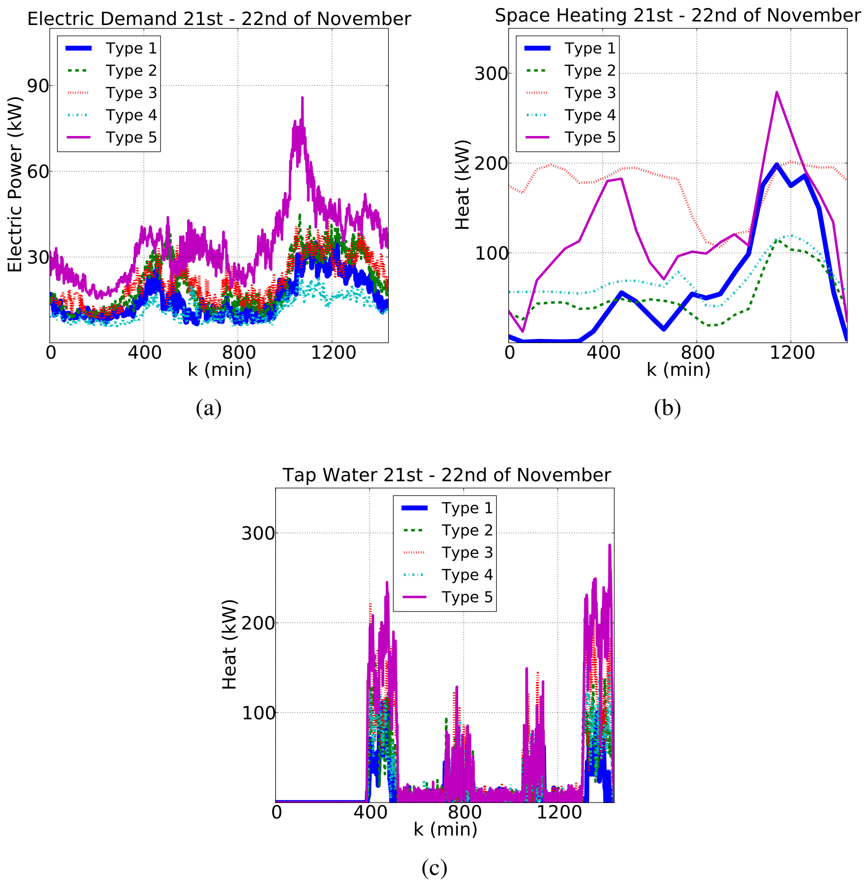
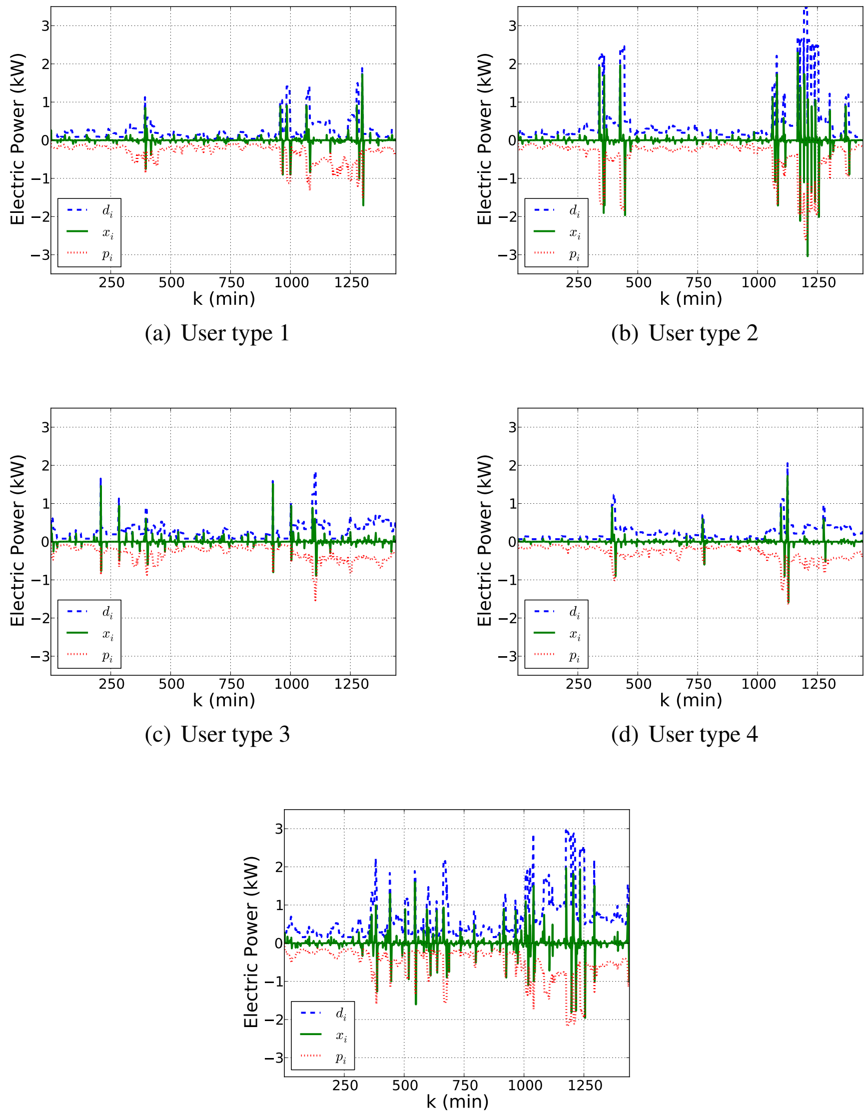
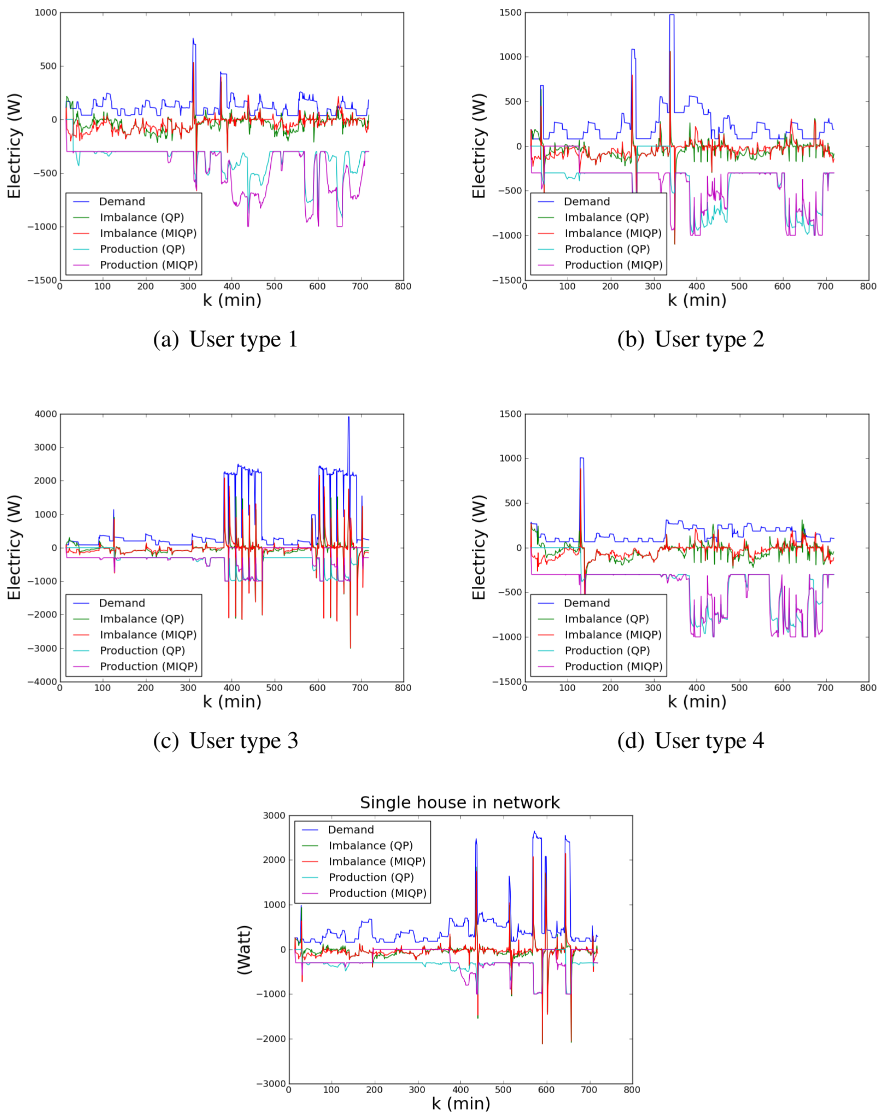


Figure A.1 – Realistic demand patterns [49]. All curves represent the sum of 50 individual users belonging to each of the five different user types.



(a) User type 1
(b) User type 2
(c) User type 3
(d) User type 4
(e) User type 5

Figure A.2 – Local patterns for five different households in the network according to Fig. 4.1 in Section 4.2.



(a) User type 1

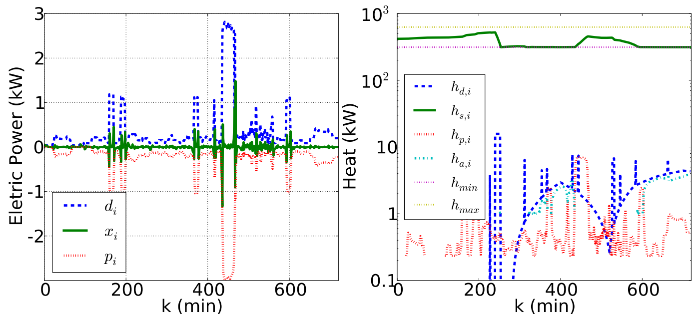
(b) User type 2

(c) User type 3

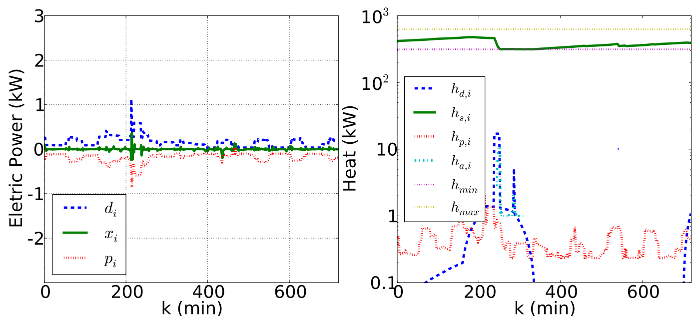
(d) User type 4

(e) User type 5

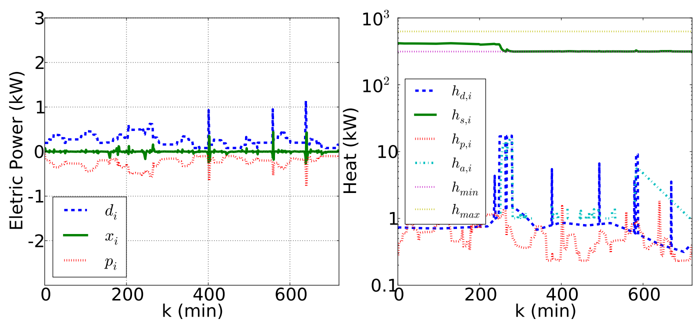
Figure A.3 – Local patterns for five different households in the network according to Fig. 4.4 in Section 4.3. The small user in (a) produces more power than needed locally, and the large user (e) produces less power than is needed locally.



(a) Agent 1, user type 1

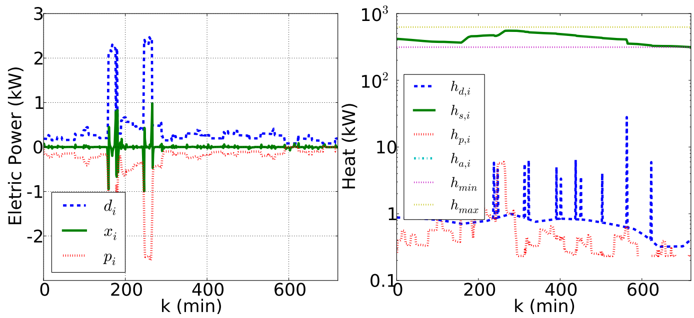


(b) Agent 2, user type 1

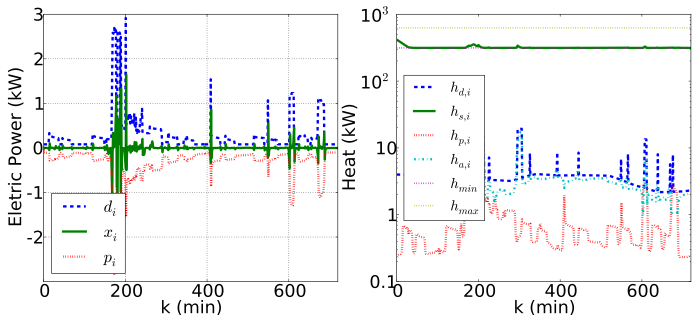


(c) Agent 3, user type 2

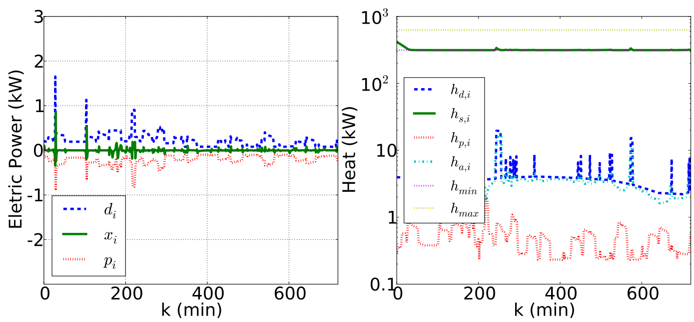
Figure A.4 – Electricity and Heat storage patterns for Chapter 5, case 4; all 10 households in the network have a 0.1-3.0kW electric μ-CHP with auxiliary burner and a 200 liter water tank for heat storage.



(a) Agent 4, user type 2

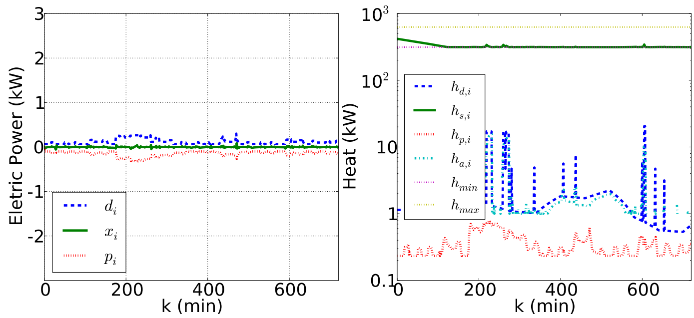


(b) Agent 5, user type 3

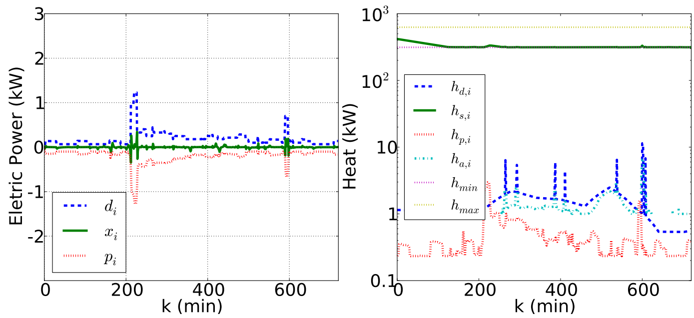


(c) Agent 6, user type 3

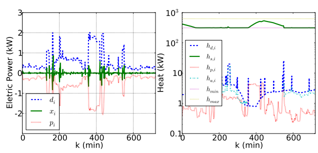
Figure A.5 – Electricity and heat storage patterns for Chapter 5, case 4; all 10 households in the network have a 0.1-3.0kW electric μ-CHP with auxiliary burner and a 200 liter water tank for heat storage.



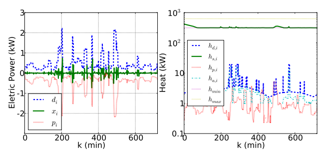
(a) Agent 7, user type 4



(b) Agent 8, user type 4

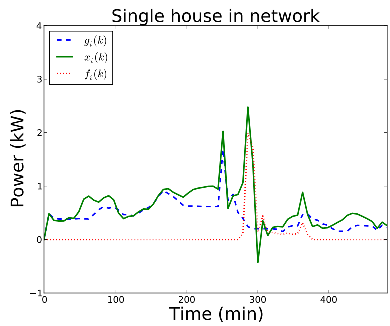


(c) Agent 9, user type 5

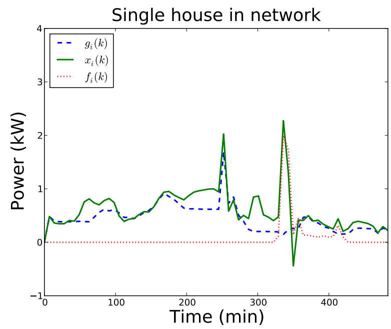


(d) Agent 10, user type 5

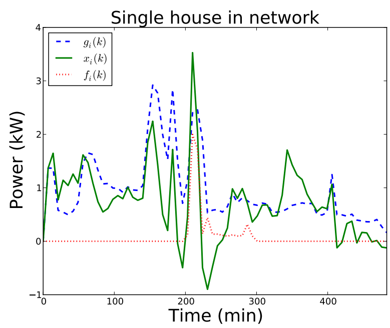
Figure A.6 – Electricity and Heat storage patterns for Chapter 5, case 4; all 10 households in the network have a 0.1-3.0kW electric μ-CHP with auxiliary burner and a 200 liter water tank for heat storage.



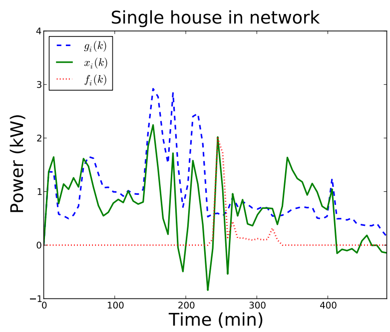
(a) User type 1. The agent assumes his own power demand stays unchanged over the prediction horizon.



(b) User type 1. Exact prediction of its own future power demand.



(c) User type 5. The agent assumes his own power demand stays unchanged over the prediction horizon.



(d) User type 5. Exact prediction of its own future power demand.

Figure A.7 – Local patterns for two households with washing machine corresponding to Chapter 6. Figures (a) and (c) show the situation when the agents do not know their future power demand exactly, while figures (b) and (d) show the situation when the agents do know their future power demand exactly. We see in (d) that the agent benefits from the exact prediction of its own future power demand, as the load from the washing machine is shifted later in time when the agents power demand is lower. These figures are also available in the master thesis [50].

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Summary

This dissertation investigates distributed decision making in an electricity network (Smart Grid) with multiple prosumers. Prosumers are both consumers and producers. The aim is to reach a global goal in the network based solely on local information. An example of such a goal is to steer the difference between electricity demand and production to a target value. We call the electricity prosumers agents, and these agents decide when to use their electrical demand and production devices. The optimal timing, for when and how much electric power to use or produce, depends on the overall power balance in the network and on the operational constraints from the devices themselves.

Algorithms are required to control the agents' decisions. One major challenge in a large-scale optimization-based control problem is to find a scalable algorithm. Therefore, this dissertation takes a distributed Model Predictive Control (MPC) algorithm based on dual-decomposition and sub-gradient iterations and adapts it to the Smart Grid. This method is expected to scale well because computations are distributed between agents. In addition, the method anticipates future situations in the network and ensures that the operational constraints are not violated.

The problems studied in this dissertation can be divided into two main branches. First, we develop an information sharing model for the power network that can be used with the distributed MPC algorithm. In the model, the agents are dynamically coupled through their power imbalance information, and the number of information neighbors per agent is a design parameter. We distinguish between a virtual information network where the agents exchange information, and the physical power grid where the agents are connected. The net mismatch of power production and consumption in the information network is imported (exported) to the overall power network, which we can view as a power reservoir. This exchange is not explicitly modeled. However, the global goal can be formulated so that the exchange of electric power with the external network (the portion of the power network that is

excluded from the information network) is minimized, or steered to a target value. It is important to notice that we work on a market level. In particular, we design a model that focuses on the amount of power that is produced and consumed, and the physics of the power network itself is not explicitly included. This is a useful approach since we can choose the virtual neighbors such that the agents trade energy with their close neighbors in the physical power grid, so that energy losses in the transmission line can be neglected. For example, when an agent's virtual neighbors are connected in the same low voltage network, the global goal can be to keep the load at the corresponding transformer station more flat.

In the second branch of problems, we test the information sharing model together with the distributed MPC algorithm in case studies with realistic data. Specifically, we study four cases; 1: the unconstrained optimal power production where the goal is to balance the demand-supply in the network, 2: the optimal power production from μ-Combined Heat and Power (CHP) systems with on-off and power modulation constraints, with the goal to balance the demand-supply in the power network, 3: the optimal power and heat production from μ-CHP systems in combination with heat-storage with the goals to cover the local heat demand and to balance the demand-supply in the network, and 4: the optimal power demand from washing machines with the goal to flatten the power demand in the network. The three last cases involve non-convex constraints due to the binary on-off decisions. This implies that the distributed MPC algorithm cannot be applied directly. Therefore, we suggest and compare two different ways to deal with this challenge. In one implementation we keep the constraints convex, which means that we do not exclude the gap between zero power output and the minimum power output from the μ-CHP in the optimization problem. If the solution of the optimization problem lies in this gap outside the physical power range of the device, the physical value that is closest to the solution of the optimization problem is implemented. The other method is to include the binary constraints explicitly in the optimization problem. We call this the mixed integers formulation. The advantage with the first implementation is that the computation is faster, while the second method results in better performance of the network in terms of the objective function.

By comparing a centralized implementation and a distributed implementation, we find that the distributed method scales better. The centralized implementation of the mixed integer formulation cannot even be solved for more than around 25 agents with the software and hardware we use. With the distributed implementation, the computation time per agent does not increase much with the network size for larger networks.

An advantage of the distributed MPC method is that it behaves like a market mechanism. The Lagrangian multipliers are interpreted as price signals (shadow prices) in economics and game theory. In our model, when there is a large change in demand or supply at one agent, the shadow price rises most at this agent and at neighbors with a short distance in the information matrix. If the imbalance is high then the shadow price rises quicker when compared to low imbalance. When there is a balance between production and demand in the network, the shadow prices do not change. The shadow prices will be highest at the prosumer where the imbalance occurred, and at his close information neighbors. This means that close-by information neighbors will have the opportunity to react the fastest to an imbalance. That the close-by information neighbors have the highest shadow price also means that they have the most to earn by producing. Thus, price signals spread as a ripple in water through the connections in the information network.

We conclude that our modelling approach along with distributed MPC is useful to coordinate local decisions in the Smart Grid. Different implementations are compared, and overall the the mixed integer implementation of the algorithm performs best with respect to the global goal, and it finds the solution well within the required time.

Samenvatting

Dit proefschrift onderzoekt gedistribueerde besluitvorming in een elektriciteitsnetwerk (Smart Grid) met meerdere electriciteitsproducenten en -consumenten. Het doel is om door middel van informatieuitwisseling de vraag naar en productie van elektriciteit door de producenten en consumenten te balanceren in onderdelen van het netwerk, bijvoorbeeld in een woonwijk. Hierbij nemen we aan dat de producenten en consumenten als actieve agents kunnen opereren en, binnen grenzen, kunnen beslissen wanneer zij energievragende of energieproducerende apparaten aan en uit zetten. Door deze beslissingen goed op elkaar af te stemmen, kan de belasting van, bijvoorbeeld, een bijbehorend transformatiestation verminderen en de energieconsumptie van het electriciteitsnetwerk als geheel afnemen.

Het is nodig om computer algoritmes te ontwikkelen die deze beslissingen coördineren, omdat het gedrag en de toestand van vele (honderden) apparaten deze beslissingen beïnvloeden. Een grote uitdaging binnen een dergelijk grootschalig optimalisatieprobleem is dan het vinden van *schaalbare* algoritmen.

Dit proefschrift onderzoekt hoe een bestaand gedistribueerd Model Predictive Control (MPC) algoritme aangepast en geïmplementeerd kan worden voor dergelijke Smart Grids. Deze methode lijkt op voorhaand goed schaalbaar, aangezien de beslissingen gedistribueerd worden berekend. Daarnaast maakt deze methode het mogelijk om te anticiperen op toekomstige situaties binnen het netwerk en rekening te houden met technische beperkingen van de agents in het netwerk. We onderzoeken de toepasbaarheid van deze MPC algoritmes voor de control van energieagents in twee stappen.

In de eerste stap ontwikkelen wij een model om informatie over het verschil tussen energieproductie en -consumptie te delen tussen de agents. Met behulp van deze informatie kan het gedistribueerde MPC algoritme het verschil tussen energieproductie en -consumptie naar de rest van het netwerk coördineren. In dit model hebben de agents geen volledige kennis van de energieverschillen in het complete

netwerk, maar wisselen alleen informatie over hun eigen verschil uit met hun directe buren. Het is belangrijk op te merken dat agents informatie uitwisselen over een virtueel informatienetwerk dat wordt bepaald door een informatiematrix. Deze matrix legt vast welke agents elkaars buren zijn, en is een ontwerpparameter binnen het model. Daaruit volgt dat dit informatienetwerk niet noodzakelijkerwijs identiek is aan het fysieke electriciteitsnetwerk. Op basis van de informatie kunnen dan de agents beslissingen nemen over het vermeerderen of verminderen van de productie of consumptie van energie. Het netto verschil tussen de energieproductie en energieconsumptie in het informatienetwerk wordt geïmporteerd (geëxporteerd) naar het gehele elektriciteitsnetwerk. Deze uitwisseling wordt niet expliciet gemodelleerd. Het doel kan echter zo geformuleerd worden dat de power uitwisseling met het externe netwerk geminimaliseerd, of richting een doelwaarde gestuurd wordt.

We combineren dit model met een gedistribueerd MPC algoritme om de optimale keuze waarde voor de energie te bepalen. Een voordeel van deze algorithme is dat het zich gedraagt als een marktmechanisme. De Lagrange multiplicatoren dat moet worden geïntroduceerd bij deze methode, worden geïnterpreteerd als prijssignalen (schaduwprijzen). Als er geen verschillen zijn tussen vraag en aanbod in het netwerk, zijn de schaduwprijzen constant. In geval er verschillen ontstaan, stijgen door onze algoritmes de schaduwprijzen het snelst bij de agenten die de grootste verandering in vraag en aanbod introduceren en bij zijn directe buren op kleine afstand in de informatiematrix. Door middel van de informatiematrix krijgen de informatieburen dan het snelst de gelegenheid om te reageren op dit verschil. Dat deze informatieburen de hoogste schaduwprijs krijgen, betekent ook dat zij het meest kunnen profiteren van het wegnemen van het energieverschil door hun productie of consumptie aan te passen. Deze aanpassingen introduceren weer verschillen bij de volgende buren, die ook weer reageren, enzovoorts. Zo verspreiden prijssignalen zich als een rimpeling door water door de verbindingen in het informatienetwerk.

In de tweede stap testen we onze modellen voor informatiedeling in samenhang met gedistribueerde MPC algoritmes in vier case studies en met realistische energieconsumptie patronen. Wij simuleren de volgende gevallen; 1: de lokale stroomproductie is onbeperkt, 2: de stroomproductie van μ-Microwarmtekrachtkoppeling (WKK) systemen met aan-uit beslissingen en beperkingen op de energiemodulatie, 3: de stroom- en warmteproductie van μ-WKK systemen in combinatie met warmteopslag, en 4: de stroomvraag van wasmachines. In alle gevallen is het doel om de electriciteitsvraag binnen het netwerk te af te vlakken. In de laatste twee gevallen moet ook aan de lokale warmtevraag voldaan worden. Verder geldt voor de laatste drie gevallen dat de beperkingen op de electriciteitsproductie niet convex is vanwege de aan-uit beslissingen. Dat heeft als gevolg dat het gedistribueerde MPC algoritme niet direct toepasbaar is. Daarom passen we het algoritme aan op twee verschillende manieren en evalueren beide varianten. In één variant behouden we de convexe beperkingen. Mocht dan de oplossing van het optimalisatieprobleem buiten het fysieke bereik van een agent liggen, gebruiken we de controlwaarden die het dichtst bij de oplossing van de optimalisatie liggen. In de andere variant modeleren we de binaire beperkingen expliciet mee in het optimalisatieprobleem. Dit resteert in een mixed integer optimalisatieprobleem. We zien in onze simulaties dat het voordeel van de eerste implementatie is dat de berekeningen sneller zijn, omdat de berekeningen (veel) eenvoudiger zijn. Terwijl het voordeel van de tweede methode is dat de prestaties van het netwerk in termen van de doelfunctie beter zijn, door beter rekening te houden met de beperkingen.

Verder vergelijken we een gecentraliseerd en een gedistribueerde implementatie van het MPC. Het blijkt dat de gedistribueerde methode beter schaalbaar is omdat benodigde rekentijd per agent niet veel toe als het netwerk groeit. De gecentraliseerde implementatie van het mixed integer probleem blijkt problematisch: voor meer dan ongeveer 25 agents is het probleem oplosbaar voor de software en hardware die wij gebruiken.

We concluderen in het algemeen dat onze modelering in combinatie met het gedistribueerde MPC algoritme goed gebruikt kan worden om lokale beslissingen in het Smart Grid te coördineren. Twee voordelen in vergelijking met andere methodes in de literatuur zijn dat we voorspellingen gebruiken en dat de methode schaalbaar is naar een grote netwerk. We adviseren om de mixed integers formulatie te gebruiken omdat dit het best presteert met betrekking tot de doelfunctie, en het vindt de oplossing ruim binnen de vereiste tijd. Het informatienetwerk moet zo gemaakt worden dat de virtuele buren energie verhandelen in hun directe omgeving in het fysieke elektriciteitsnet. Zo kan bijvoorbeeld de belasting van het bijbehorende transformatiestation vlak worden gehouden als een agent zijn virtuele buren verbonden zijn in hetzelfde laagspanningsnet.

About the Author

Gunn Kristine Holst Larsen received her B.S. degree in Physics from University of Oslo, Norway, in 2006, and one of the semesters were completed as an exchange student at the University of British Columbia, Canada. During the summer 2006 she received a summer student grant at the Center of Materials Science and Nanotechnology, Oslo, Norway, to work on solar cells. She obtained her M.S. degree in Experimental Particle Physics from University of Oslo, Norway, in 2008, and this included several visits to the European Organization of Nuclear Research (CERN).

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